Basel Committee
on Banking Supervision

Consultative Document

Operational risk – Revisions to the simpler approaches

Issued for comment by 6 January 2015

October 2014

BANK FOR INTERNATIONAL SETTLEMENTS

This publication is available on the BIS website (www.bis.org).

ISBN 978-92-9131-868-1 (print)

ISBN 978-92-9131-869-8 (online)

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Executive summary

Introduction

In the wake of the financial crisis, the Basel Committee on Banking Supervision has been reviewing the adequacy of the capital framework. The aim is not only to address the weaknesses that were revealed during the crisis, but also to reflect the experience gained with implementation of the operational risk framework since 2004. At that time, the Committee made clear that it intended to revisit the framework when more data became available. Despite an increase in the number and severity of operational risk events during and after the financial crisis, capital requirements for operational risk have remained stable or even fallen for the standardised approaches. This indicates that the existing set of simple approaches for operational risk – the Basic Indicator Approach (BIA) and the Standardised Approach (TSA), including its variant the Alternative Standardised Approach (ASA) – do not correctly estimate the operational risk capital requirements of a wide spectrum of banks.

The weaknesses of these simpler approaches stem mainly from the use of Gross Income (GI) as a proxy indicator for operational risk exposure, based on the assumption that banks' operational risk exposure increases linearly in proportion to revenue. This assumption usually turns out to be invalid. In particular, where a bank experiences a decline in its GI due to systemic or bank-specific events including those involving operational risk losses, its operational risk capital falls when it should be increasing. Moreover, the existing approaches do not take into account the fact that the relationship between the size and the operational risk of a bank does not remain constant or that operational risk exposure increases with a bank's size in a non-linear fashion. In addition, the changing operational risk profiles of banks may render a calibration based on the past behaviour of variables unfit for the future. Proxy-based indicators used in the operational risk approaches and the calibration of the associated parameters should therefore be periodically tested to ensure their continued validity. Such a review is all the more important given the lack of relevant operational risk data and experience in operational risk modelling when the original framework was designed in the early 2000s. We now have not only a richer data set to support the quantitative analysis, but also almost a decade of experience with implementation of the framework.

The Committee has therefore undertaken a fundamental review of the simpler approaches for operational risk based on extensive data relating to operational risk losses and exposure indicators from a wide range of banks. These data were assembled in several exercises, including the 2008 Loss Data Collection Exercise, the 2010 Quantitative Impact Study (QIS) and, more recently, specific collections on operational risk losses and candidate proxy indicators based on supervisory reports and other sources available to the Committee's members. Another loss data collection effort (the new QIS) is under way in parallel to this consultation, the results of which will be used to validate the proposals outlined in this paper.

The Committee's preliminary findings, based on the existing data, indicate that the current standardised framework comprising the BIA, TSA and ASA is on average undercalibrated, especially for large and complex banks, and that Advanced Measurement Approaches (AMA) capital charges are often benchmarked against this undercalibrated capital requirement. Reflecting this concern, the revised Standardised Approach (SA) attempts to improve the calibration while addressing the weaknesses of the existing approaches identified above.

Main elements of the revised Standardised Approach (SA)

The review seeks to address the weaknesses identified in the existing approaches by (i) refining the operational risk proxy indicator by replacing GI with a superior indicator; and (ii) improving calibration of the regulatory coefficients based on the results of the quantitative analysis. During the course of the analytical work carried out over the past two years, it became apparent that: the original Basel II business lines did not differ significantly in terms of their operational risk profiles; the size of a bank was a dominant factor in operational risk exposure; and refinements to the proxy indicator could enhance risk sensitivity. It was therefore considered appropriate to develop only one approach based on a single indicator of operational risk exposure with size-based coefficients. A single non-model-based approach also addresses the Committee's objectives of promoting simple and comparable approaches while still maintaining risk sensitivity.

Refinement of the proxy indicator for operational risk

The Committee investigated more than 20 potential benchmarks for their sensitivity to operational risk exposure. In this exercise, the Committee considered in addition to statistical analysis the economic reasoning behind various potential indicators. Most of the potential indicators of operational risk exposure evaluated by the Committee relate to balance sheet items and income statements. The financial statement-based proxies for operational risk fall broadly into two categories – (i) proxies based on assets and liabilities and (ii) proxies based on items of income and expenditure. While the proxies based on assets and liabilities would, to a great extent, avoid the cyclicality associated with the proxies based on income and expenditure, they face a major limitation in their inability to capture off-balance sheet or fee-based businesses, and they are affected by valuation and accounting practices. On the other hand, measures based on income and expenditure provide various possibilities to explore. Therefore, the Committee's analysis focused on the latter set of indicators.

Based on the qualitative and quantitative analysis, the Committee has identified the Business Indicator (BI) as the most suitable replacement for GI, as it addresses most of the latter's weaknesses. The BI comprises the three macro-components of a bank's income statement: the "*interest component*", the "*services component*", and the "*financial component*". The BI's power, as compared with GI and other potential indicators, lies in its superior ability to capture a bank's exposure to the operational risk inherent in a bank's mix of business activities.

The BI includes items sensitive to operational risk that are omitted or netted from the GI definition. In addition, the BI uses the absolute values of its components, thereby avoiding counterintuitive results based on negative contributions of components to capital charges from net losses under the existing framework. Moreover, the BI reduces the relative weight or contribution of components of the financial statement that are associated with activities traditionally less exposed to operational risk, and increases that of the components associated with activities more closely associated with operational risk (eg gains and losses on traded and sold portfolios, commissions from services payments, fees received from securitisation of loans and origination and negotiation of asset-backed securities, penalties from mis-selling and inadequate market practice). Many of these components proved to be at the core of the financial crisis.

The increased effectiveness of the BI as a proxy is also well supported by statistical analysis. In order to test the power of proxy indicators to predict operational risk capital exposures, it was necessary to relate them to some measure of capital requirement based on operational loss experience. The Committee has therefore developed a quantitative model based on bank-internal loss data. Known as the Operational risk Capital-at-Risk (OpCaR) model, this tool can be used to estimate hypothetical capital requirements reflecting own-loss experience. Internal loss experience as the sole basis for the calibration of the revised SA was considered appropriate, as other data elements that constitute part of the AMA – such as scenarios – are not readily useable for a regulatory model that should be applicable globally.

In particular, the estimates of industry average operational risk capital exposure obtained from the OpCaR model have been used for two purposes: (i) to inform the regressions undertaken to assess which operational risk proxy indicator (eg the BI, GI, Total Assets etc) best correlates with the capital needs of banks; and (ii) to calibrate the coefficients to be applied to the new BI proxy indicator selected for the revised SA.

Improving calibration of the regulatory coefficients

The recalibration of the current regulatory coefficients in the BIA (alpha) and TSA (betas) focused on three aspects: (i) the review of adequacy of operational risk capital levels; (ii) re-assessment of the need for having different regulatory betas based on the business lines; and (iii) introduction of the new size-based regulatory coefficients.

The Committee estimated the amount of capital required to fully cover the exposure to operational risk as reflected in loss experience and the pertinent OpCaR. Apart from the undercalibration revealed by the analysis, it was observed that capital needs for operational risk increase in non-linear fashion with the bank's size, suggesting the need to introduce a set of different coefficients based on the size of the bank as reflected in the value of the BI. The analysis of the current TSA using the OpCaR model revealed that the regulatory business lines do not differ statistically in terms of riskiness when the riskiness is measured by coefficients applied to the proxy indicator apportioned between the business lines. A similar outcome was obtained by industry studies, which were presented to, and discussed with, the Committee.

The preliminary calibration has identified a [five]-bucket structure with coefficients increasing in value from [10%] to [30%] with the rise in the value of the BI. The number and width of the buckets, as well as the corresponding coefficients values indicated in this document, represent tentative conclusions and will be refined based on the data collected as part of the ongoing QIS exercise.

Dealing with banks facing specific situations

Banks with very high net interest margin (NIM)

Net interest income remains the BI's dominant component. It has been observed that bank business models in some jurisdictions emphasise high net interest income and, similarly, high net interest margin or NIM (defined as net interest income divided by interest-earning assets), and the NIM may also vary significantly across banks and jurisdictions. As a result, in extreme cases the BI may not be a proper proxy for operational risk exposure. This problem under the revised SA is similar to that faced by such banks under the TSA.

To address the issue of high interest margins, Basel II authorised the replacement of GI by an asset-based proxy (loan and advances multiplied by a fixed m-factor of 0.035) in two business lines (retail and commercial banking) under the ASA. The new QIS exercise will be used to identify alternative solutions for this issue. One possible solution would be to apply a "cap" to the NIM by normalising the interest component included in the BI downwards. A similar treatment, a floor, could be applied in the case of low interest income, thereby introducing a "boundary range" beyond which adjustments to normalise the interest component would be made.

Other issues

A small number of banks which are highly specialised in fee businesses have been identified as facing a disproportionately high capital impact under the BI. The problem stems from the fact that the structure of the BI, which was designed to capture the operational risk profile of a universal bank, may not apply accurately to banks engaged predominantly in fee-based activities. The Committee will respond to this issue if it is further evidenced by the results of the new data collection exercise.

Risk management expectations under the revised SA

In the current operational risk regulatory framework, adoption of the TSA/ASA is subject to supervisory approval as well as to compliance with certain explicit qualifying criteria for risk management. As the revised SA will become the "entry level" capital methodology, its use will not require supervisory approval nor will it be accompanied by any explicit risk management criteria. This does not mean, however, that the revised framework is less rigorous than the existing one, as this would not be appropriate in the light of the substantial operational risk losses incurred by banks during and after the recent financial crisis.

The Committee's *Principles for the Sound Management of Operational Risk* (PSMOR or the "Principles") set expectations for the management of operational risk. All internationally active banks should implement policies, procedures and practices to manage operational risk commensurate with their size, complexity, activities and risk exposure, and seek continuous improvement in these areas as industry practice evolves. In order to enhance operational risk management, the Principles provide comprehensive guidance regarding the qualitative standards that should be observed by large internationally active banks. The Committee considers it appropriate to achieve more definitive, rigorous and comprehensive implementation of the Principles by setting out specific guidance under Pillar 2 to be observed by large internationally active banks.

Next steps

The Committee welcomes comments from the public on all aspects of this consultative document. Comments on the proposals should be uploaded by 6 January 2015 using the following link: http://www.bis.org/bcbs/commentupload.htm. All comments will be published on the website of the Bank for International Settlements unless a respondent specifically requests confidential treatment.

Once the Committee has reviewed responses to this consultative document and the results of the QIS, it intends to publish the final standard within an appropriate timeframe and provide sufficient time for implementation. Before the final standard is published, implementation arrangements (including the timetable) will be discussed by the Committee, taking into account the range of other reforms that have been, or are due to be, considered by the Committee.

Revisions to the operational risk standardised approaches

I. Background

Current approaches for the measurement of operational risk

1. The Basel framework provides three approaches for the measurement of the capital charge for operational risk. The simplest is the Basic Indicator Approach (BIA), by which the capital charge is calculated as a percentage (alpha) of Gross Income (GI), a proxy for operational risk exposure. Being the most basic approach, its adoption does not require prior supervisory approval. The most advanced methodology is the advanced measurement approaches (AMA), which allows banks to use internal models to calculate their capital requirements. Adoption of the AMA requires prior supervisory approval and involves implementation of a rigorous risk management framework. The third approach, the Standardised Approach (TSA), which is positioned as an intermediate approach between the BIA and the AMA, requires banks to divide their total GI into eight business lines and to calculate capital requirements as a sum of the products of the GI attributed to each business line and the specific regulatory coefficients (betas) assigned to each line. Since the adoption of the TSA requires compliance with a set of qualitative criteria relating to operational risk management systems, banks are required to obtain prior approval from their supervisory authorities before moving to this approach. A variant of the TSA, the Alternative Standardised Approach (ASA), allows banks with high interest margins to calculate their operational risk capital requirements by replacing the GI for two business lines – retail banking and commercial banking – with a fixed percentage of their loans and advances. Adoption of the ASA is allowed by the respective supervisory authorities at their national discretion.

Rationale for the review

2. In the wake of the financial crisis, the Committee has been reviewing the capital adequacy framework with a view not only to addressing the weaknesses that were revealed during the crisis, but also to reflect, in general, the implementation experience gained since 2004, when Basel II was introduced. At the time of implementation, the Committee made clear that it intended to revisit the operational risk framework when more risk-sensitive data become available. Despite an increase in the number and severity of operational risk events during and after the financial crisis, capital requirements for operational risk have remained stable or even decreased for the standardised approaches, calling into question their effectiveness and calibration. Some of these events have even threatened to precipitate bank failures.

3. The existing set of simpler approaches (BIA/TSA/ASA) fails to correctly estimate the operational risk capital requirements of a wide spectrum of banks. These approaches are based on the use of GI as a proxy indicator for operational risk exposure. The BIA is based on the assumption that banks' operational risk exposures increase linearly in proportion to revenue. The other two approaches seek to refine this approach by distinguishing eight business lines (TSA) or by introducing an assets-based proxy indicator in two of the eight business lines (ASA).

4. During the financial crisis, bank financial performance resulting in lower GI challenged the underpinning assumption that revenue, and more specifically GI, was the most effective proxy for operational risk. The most common situation involved banks experiencing a decline in their GI due to systemic or bank-specific events, including those involving operational risk losses, and seeing a commensurate decline in operational risk capital when intuitively this should have either stayed at the same level or increased. Moreover, the sensitivity of operational risk exposure can vary with the size of a

bank and changes constantly. In the case of the TSA, concerns were already emerging at the time of its finalisation that the regulatory business lines might not differ statistically in terms of riskiness to the extent indicated by the different coefficients applied to the proxy indicators for those business lines. Therefore, the proxy-based indicators used in the current operational risk approaches and the calibration of the associated coefficients need to be periodically tested to ensure their continued validity. However, over the past decade, no rigorous review has been made of the effectiveness of GI (or other potential indicators) as the proxy for the operational risk exposure of a bank and the adequacy of the calibration of the regulatory coefficients of the BIA and TSA. Such a review is considered all the more important in view of the lack of both relevant operational risk data and experience in operational risk modelling when the original framework was designed in the early 2000s.

5. Over the past few years, significant progress has been made across the industry with respect to the quality of banks' internal processes for the identification and collection of operational risk losses. We now have both post-Basel II data and data through the crisis period to support a quantitative analysis. We also have almost a decade of framework implementation experience. Together with the expanded data set, this experience allowed us to test the performance of the framework and review its calibration across a larger portion of the business cycle, including a stressed period. The current review is based on an extensive set of operational risk losses and exposure indicators from a wide range of banks in several data collection exercises, including the 2008 Loss Data Collection Exercise, the 2010 Quantitative Impact Study (QIS), and, more recently, specific collections on operational risk losses and candidate proxy indicators based on supervisory reports and other sources available to the Committee.[1] A new QIS loss data collection effort is under way in parallel to this consultation, the results of which will be used to validate and finalise the proposals.

Q1. Are there any other weaknesses in the existing set of simple approaches that should be addressed by the Committee?

II. Principles of the revised Standardised Approach (SA)

6. While the work concerning review of the simpler approaches was motivated primarily by the need to address the weaknesses described above, the Committee is mindful of the need to ensure that the framework should be risk-sensitive but simple, and that capital outcomes should be comparable across banks. The notion of simplicity concerns both the simplicity of the capital calculation process and the rule text. The concept of comparability of capital outcomes addresses the principle that similar risk profiles should attract similar risk weights across banks and jurisdictions. In the revised SA, these objectives have been reflected appropriately.[2] In particular, the following principles were kept in view while formulating the revised SA:

[1] The 2008 LDCE collected information, among other things, on banks' internal loss data, while the 2010 QIS gathered information at both the banking group and business line levels on balance sheet and income statement items (including GI and its components), and on the aggregate number and amount of operational risk losses above specific thresholds.

[2] Basel Committee on Banking Supervision, *The regulatory framework: balancing risk sensitivity, simplicity and comparability,* Discussion Paper, July 2013.

- There should be only one simple approach given the need to ensure simplicity and comparability of outcomes in the framework;
- The approach should address the known weaknesses of the existing simpler approaches while retaining the fundamental attributes of the current framework;
- It should be simple enough to understand, not unduly burdensome to implement, should not have too many parameters for calculation by banks and it should not rely on banks' internal models;
- It should exhibit enhanced risk sensitivity;
- It should be calibrated according to the operational risk profile of a large number of banks of different size and business models; and
- It should be suitable for implementation across a wide range of jurisdictions and banks.

III. Elements of the revised SA

Objectives of the review

7. The review's primary objective is to address the weaknesses identified in the existing approaches. These are addressed by a two-pronged strategy that consists of (i) refinement of the proxy indicator for operational risk exposure (replacement of GI by a superior indicator); and (ii) improving the calibration of the regulatory coefficients based on the results of the quantitative analysis.

8. During the course of the analytical work carried out over the past two years, it became apparent that the business lines did not differ significantly in terms of their operational risk profiles when measured by portions of a proxy indicator multiplied by an associated coefficient. In addition, a bank's size was seen to be a dominant factor in distinguishing its operational risk exposure. It was therefore considered appropriate to develop only one approach based on a single indicator of operational risk exposure with size-based coefficients instead of the business line-based beta factors. A single non-model approach also meets the Committee's objective of simplicity and helps ensure comparability of capital outcomes.

9. The revised SA seeks proportional implementation of the Principles for the Sound Management of Operational Risk (PSMOR or "Principles") by all banks based on size, complexity, and the nature of activities, including more rigorous and comprehensive implementation of the Principles by larger banks.

Q2. Does a single standardised approach strike an appropriate balance across the Committee's objectives of simplicity, comparability and risk sensitivity?

Refinement of the proxy indicator of operational risk exposure

Investigation of potential indicators of operational risk exposure

10. To identify an operational risk indicator more suitable than GI, the Committee investigated more than 20 potential benchmarks for their sensitivity to operational risk. In this exercise, the Committee utilised statistical analysis and considered the economic reasoning of various potential indicators.

11. A majority of the potential indicators of operational risk exposure evaluated by the Committee relate to balance sheet and income statement items. The indicators were selected for investigation bearing in mind that these should be straightforward to implement (readily available, and commonly standardised to reduce the room for gaming/regulatory arbitrage), easy to calculate (so as to limit implementation burden), and capable of addressing potential inconsistencies and weaknesses of the current regime. Finally, they should also be intuitive and have clear economic significance.

12. Financial statements-based proxies for operational risk may be based on either (i) assets and liabilities, or (ii) income and expenditure items. While proxies based on assets and liabilities, to a great extent avoid the cyclicality associated with proxies based on income and expenditure, they may fail to capture operational risk associated with off-balance sheet or fee-based businesses, and they are affected by valuation and accounting practices. On the other hand, measures based on income and expenditure are more cyclical, but capture the non-balance-sheet nature of operational risk and, in particular, recognise the non-interest revenue and expense aspects of bank operations.

Replacement of Gross Income with the Business Indicator

Working definition of the Business Indicator

13. The BI is based on the three macro-components of a bank's income statement: the "interest component", the "services component" and the "financial component" (please see **Annex 1** for details of the various items included in these components).

Business Indicator = Interest component + Services Component + Financial component

where,

Interest component =	Absolute value (Interest Income – Interest Expense)
Services component =	Fee Income + Fee Expense + Other Operating Income + Other Operating Expense
Financial component =	Absolute value (Net P&L on Trading Book) + Absolute Value (Net P&L on Banking Book)

Rationale for the construction of the BI

14. In determining the appropriateness of income- and expenditure-based proxies for operational risk exposure, three important aspects need to be considered.

15. *First*, it is possible to use different combinations of items of income and expenditure with and without taking their absolute values: (i) total income/expenditure, (ii) select items of income/expenditure which exhibit sensitivity to operational risk exposure, (iii) sum of income and expenditure items, and (iv) net profit (income – expenditure).

16. *Second*, the income and expenditure items of a typical financial institution span three broad components: (i) *Interest component (*interest income from loans and debt securities and interest expenses incurred on deposits that fund these loans and own debts), (ii) *Services component* (fees earned on services rendered and fees paid on services used), and (iii) *Financial component (*net profit or loss on the trading and banking books).

17. *Third*, operational risk exposures may arise from any type of operation, whether or not these generate income or incur expenses. In general, therefore, a measure based on summing items of income and expenditure should perform better than one that nets these items. However, in some limited areas, summation of income and expenditure may prove more prone to cyclicality than net income, particularly when income and expenses are highly correlated.

(a) Interest component

18. The Committee explored the possibility of using the sum of interest income and interest expense as a measure of operational risk exposure. However, it was observed that changes in interest rate levels would render this measure highly cyclical when such changes do not necessarily imply a corresponding change in the operational risk exposure. In particular, countries where interest rates are volatile would see their operational risk capital requirements vary significantly over the interest rate cycle. Also, since interest rates vary significantly across countries, a common calibration based on the sum of interest income and expense would result in an unduly high capital charge for banks in jurisdictions with high interest rates.[3] On the other hand, interest margins are more stable.

19. In this light, the Committee explored the effectiveness of net interest income as a proxy for operational risk exposure in this business segment. On average, net interest income proved a better measure of operational risk exposure than the sum of interest income and expense. To prevent any net loss incurred by lending activities from reducing the operational risk charge, the Committee also proposes to use absolute values for the net interest income.

(b) Services component

20. The sum of fee income and expenses within the services component exhibits stable behaviour over time. Unlike in the case of lending business, where funding and investing are closely tied to each other, there is no comparable relationship between services offered and services used. Here, the sum of fee income and expenses better captures a bank's operational risk in services activities, while netting would result in an underestimation of the scale of operations.

(c) Financial component

21. In the case of the financial component, it would not make a difference if the calibration were based on the sum of absolute values of gains and losses or the absolute values of net P&L from the trading and banking book activities, as both measures are generally unaffected by cyclical changes in the economy. However, since gains and losses in the trading and banking books are typically reported in financial statements on a net basis, it would be more practical to use net gains or losses instead of summing up all gains and all losses separately and adding them together. In addition, taking the absolute values of net P&L in the trading and banking books prevents net losses in these books from reducing capital requirements.

[3] For a given operational risk profile, banks in a high interest rate environment (and consequently those with a high sum of interest income and interest expenses) should have a low coefficient value. However, a coefficient based on a common calibration will be higher than required for these banks and result in a higher than required capital charge. The opposite will be true for banks operating in a low interest rate environment.

Comparison between the BI and GI

BI and GI decomposed by macro-components — Table 1

Component of a bank's income statement	Gross Income items	Business Indicator items
Interest	Interest Income – Interest Expense	Absolute value (Income – Expense)
Services	Fee Income – Fee Expense + Other Operating Income	Fee Income + Fee Expense + Other Operating Income + Other Operating Expense
Financial	Net P&L on Trading Book	Absolute value (Net P&L on Trading Book) + Absolute Value (Net P&L on Banking Book)
Other	Dividend Income	Not included

22. The BI's enhanced predictive power, as compared with that of GI and other potential indicators, stems from its ability to capture a bank's volume of business, hence the associated operational risk. Keeping the current GI and adjusting the way it enters into the regulatory capital calculation (eg by excluding negative years from the three-year average or by setting a floor for its components) has proved less satisfactory. The BI improves upon GI in certain important respects, as it:

- includes items sensitive to operational risk, which are omitted or netted from the GI definition (eg P&L from the banking book, other operating expenses, fee and commission expenses);
- avoids counterintuitive results (eg negative contributions to the capital charge from net trading losses);
- reduces the weight of components that are associated with activities traditionally regarded as less exposed to operational risk (eg interest income generated by pure lending activity); and
- increases the weight of components associated with activities more closely related to operational risk (eg gains and losses on traded or sold portfolios, commissions from services payments, fees received from securitisation of loans and origination and negotiation of asset-backed securities, penalties from mis-selling and inadequate market practice) – many of which were at the core of the financial crisis.

Statistical support for the BI

23. The BI was identified as the most effective indicator in a rigorous statistical analysis. The explanatory power of the candidate indicators was assessed by assuming non-linear as well as linear relationships between the operational risk exposure and these indicators. The BI proved significantly more risk-sensitive than GI or other proxies, such as total assets, administrative costs and total provisions according to statistics and accuracy measures appropriately selected for this purpose. In addition, it showed satisfactory robustness and stability over time.

24. The risk sensitivity or the explanatory power was the most important property considered in the selection of proxy indicators. In order to test the explanatory power of proxy indicators, it is necessary to relate them to some measure of the operational risk exposure. The Committee developed a quantitative model based on a bank's internal loss data, the "Operational risk Capital-at Risk (OpCaR) model" that could be used to determine its hypothetical capital requirement reflecting its own loss experience.

25. Internal loss experience was considered appropriate as the sole basis for the calibration of the revised SA as other data elements that are part of the AMA – such as scenarios – are not readily useable for a regulatory model that should be applicable globally.

26. The OpCaR builds on theories and approaches commonly used in actuarial science and is similar to the internal models most widely used by banks for operational risk management: the loss distribution approaches (LDA). Using aggregated internal loss data as an input to estimate the frequency and severity distributions of losses, it obtains the desired measure of risk at the 99.9th percentile of the frequency-severity compound distribution. Estimates obtained from the OpCaR model have been used for two purposes: (i) to inform the relative performance of candidate proxy indicators (eg the BI, GI, Total Assets etc) in terms of risk sensitivity; and (ii) to calibrate the coefficients to be applied to the proxy indicator selected for the revised SA. The OpCaR methodology is described in detail in **Annex 2**. **Annex 3** describes the technical analysis of the 20 indicators.

Q3. Are there any further improvements to the BI that should be considered by the Committee?

Improving calibration of the regulatory coefficients

27. The calibration of the revised SA focused on three aspects: (i) review of the adequacy of operational risk capital levels, (ii) re-assessment of the need for having different regulatory betas based on business lines, and (iii) introduction of the new size-based regulatory coefficients. The calibration exercise was based primarily on the 2010 QIS data and additional data taken from supervisory reports collected by the Committee in 2012.

Adequacy of operational risk capital levels

28. The Committee estimated the amount of capital required to fully cover the exposure to operational risk of individual banks in the sample as measured by the OpCaR model. The analysis revealed that banks' operational risk capital levels under the current Basel framework were on average already undercalibrated in 2009, the year for which the data were collected under the Committee's 2010 QIS.

29. Moreover, capital needs for operational risk were found to be increasing in a non-linear fashion with a bank's size. The Committee, therefore, believes that a bucketing approach and coefficients that increase according to a bank's size would better reflect banks' operational risk profiles and associated capital needs. In particular, the capital shortfall based on the OpCaR analysis at the end of 2009 for banks that would be positioned in the largest bucket were estimated at up to 100% of the regulatory capital held for operational risk. The extent of undercalibration appears to have further increased in 2010 and 2011 as a consequence of the operational risk losses that occurred during and after the financial crisis. Conversely, smaller banks were likely to have been overcapitalised, reflecting again the non-linear relationship between banks' size and their operational risk exposure.

Re-assessment of the need for business line-based coefficients under the TSA/ASA

30. The analysis of the current TSA revealed that the Basel business lines do not differ statistically in terms of riskiness when the riskiness is measured by coefficients applied to the proxy indicator apportioned between the business lines. A similar result was obtained by industry studies.

31. In particular, the Committee's analysis showed that the required range of estimated betas under the TSA was much wider than that envisaged by the current framework. Also the ranking of the riskiness of business lines implied by the current framework appear to be flawed, as some business lines with the lowest betas of 12% experience very high operational risk losses and other business lines with mid-range betas of 15% show relatively lower levels of operational risk losses. In addition, some business lines with the highest beta of 18% (eg Trading & Sales or Payments & Settlements) showed regulatory capital

levels below the amount of the OpCaR or even the reported losses per business line. This was a result of lower GI in periods where operational losses were stable or increasing.[4]

32. Finally, the current definition of the business lines within the framework seems no longer suitable for many banks in light of the emergence of new products (eg derivatives, EFTs) and marketing channels (eg internet, mobile banking) that are changing the way business lines are organised and managed.

Determining an appropriate number of buckets and corresponding coefficients

33. The analysis demonstrated that the relationship between operational risk exposure and size increases in a non-linear fashion, suggesting the need to introduce a set of escalating coefficients based on the size of the bank as reflected in the value of the BI. To keep the framework as simple as possible, a discrete structure for the coefficients is proposed. The coefficients have been determined based on the analysis first conducted on the 2010 QIS data followed by an update reflecting data made available in 2012. The reference variable investigated in this work was the OpCaR/BI ratio, a direct estimate of the regulatory coefficient, for which banks' average figures were obtained for the whole sample of banks and for each bucket.

34. The appropriate number of buckets was determined based on a technical analysis that applies a smoothed unexpected loss (UL) function[5] to the range of BI levels. A cluster analysis was then carried out on the UL smoothing function with the aim of (i) aggregating in the same bucket banks showing a similar risk profile; and (ii) identifying the most appropriate number of buckets for the sample. The analysis identified a discrete structure for the coefficients based on the level of the BI (€100m, €1bn, €3bn, and €30bn) as shown in Table 2. The structure of the buckets, as well as the corresponding coefficients values indicated below, should be considered as preliminary and will be refined based on the new QIS exercise. Particular caution should be exercised for lower buckets where data availability for estimation was less abundant.

The proposed coefficients per bucket under the SA

Table 2

BI (€ millions)	Coefficient
0–100	[10%]
>100–1,000	[13%]
>1,000–3,000	[17%]
>3000–30,000	[22%]
>30,000	[30%]

Application of the regulatory coefficients: a layered approach

35. Applying the coefficient to the full amount of the BI introduces undesirable "cliff effects" when a bank migrates from one bucket to another. While the problem of cliff effects is not unique to operational

[4] This was particularly evident for the Trading & Sales business line, which in some cases showed very low or even negative income due to the effects of the financial crisis on the results from trading activities.

[5] The UL function was derived from the OpCaR figures, by considering only their unexpected loss (UL) component, which was compared with the BI. The function parameters were estimated by minimising a kind of adjusted R-squared adapted to the specific framework. The UL function with an associated parameter estimate was then adopted to identify relevant buckets of the BI.

risk, it is arguably more significant in the operational risk SA context than in other elements of the capital framework due in large part to the non-proportionality reflected in the coefficient values.

36. Under the "layered approach", the coefficient for a given bucket as indicated in Table 2 will be applied in a marginal manner only to the incremental portion of the BI that falls in that bucket.[6] The total operational risk capital charge for a bank will be the sum of the incremental capital charges ascribed to each of the relevant buckets. This layered approach delivers a smooth increase of capital charges with increasing values of the BI, thereby avoiding cliff effects.

37. The effective value of the coefficient for a bank under the layered approach will be somewhere between the coefficients of the lower and the upper buckets. The coefficients and the "effective" coefficients are illustrated in Table 3.

The proposed coefficients and the range of "effective" coefficients per bucket under the SA

Table 3

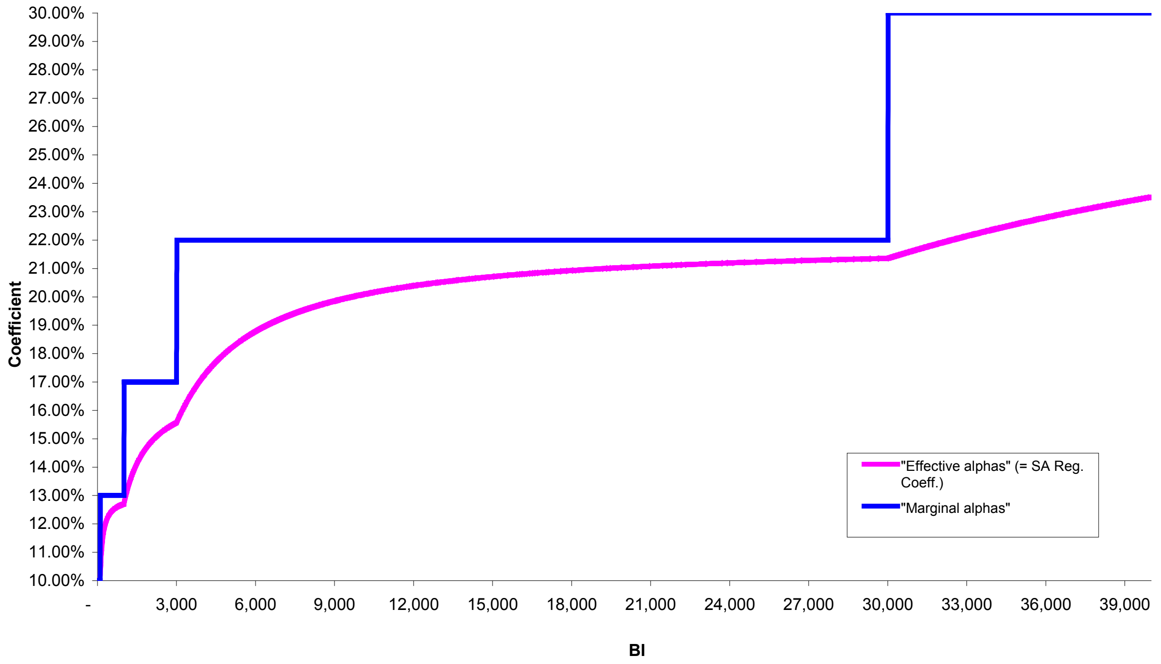
BI (€ millions)	Coefficient	Range of "effective" coefficients within a bucket
0–100	10%	10%
>100–1,000	13%	10%–12.7%
>1,000–3,000	17%	12.7%–15.57%
>3,000–30,000	22%	15.57%–21.36%
>30,000	30%	21.36%–30% (approx)

38. The "effective" coefficient values indicated in Table 3 are meant only to convey an idea of the smoothing of applicable coefficient that takes place under the layered approach. Banks will not be required to calculate the coefficients and use them in the capital calculation process, which remains much simpler, as illustrated by the following numerical examples based on Table 2. Figure 1 shows a plot of the proposed coefficients and corresponding "effective" coefficients under the SA.

<u>Bank</u>	<u>BI</u>	<u>Capital calculation</u>
A.	80	80*10% = 8
B.	800	100*10% + 700*13% = 101
C.	2,000	100*10% + 900*13% + 1,000*17% = 297
D.	20,000	100*10% + 900*13% + 2,000*17% + 17,000*22% = 4,207
E.	40,000	100*10% + 900*13% + 2,000*17% + 27,000*22% + 10,000*30% = 9,407

[6] For example, for a bank with a BI of €500, the coefficient of 13% will be applied to an amount of 400 (= 500 – 100) of the BI.

Figure 1: Plot of the proposed coefficients and "effective" coefficients under the SA



Q4. What additional work should the Committee perform to assess the appropriateness of operational risk capital levels?

Q5. Are there any other considerations that should be taken into account when establishing the size-based buckets and coefficients? How many BI buckets would be practical for implementation while adequately capturing differences in operational risk profiles?

Q6. Are there any other considerations that should be taken into account when replacing business lines with size-based buckets?

Q7. Could there be any implementation challenges in the proposed layered approach?

Calculation of minimum capital requirements

39. The revised SA is based on two inputs – (i) the BI, and (ii) the regulatory coefficients applied in a layered manner. Banks using the SA must hold capital for operational risk calculated according to the following formula.

$$K_{SA} = [\Sigma_{years1\text{-}3}\, \Sigma\, (BI_j \times \alpha_j)]/3$$

where

K_{SA} = the capital charge under the revised SA

BIj = annual value of the BI apportioned to bucket "j" (1…n) in a given year

α_j = coefficient for bucket "j"

Dealing with banks facing specific situations

Banks with very high or very low net interest margin (NIM)

40. The NIM is usually the dominant component in the BI. Normally the NIM fluctuates, and averaging the BI for three years for the purpose of calculating the capital charge for operational risk smoothens the impact of these fluctuations. However, if the NIM is structurally and persistently very high or very low, this could result in considerable overestimation or underestimation of the operational risk capital requirements.

41. Banks in some jurisdictions may emphasise a high NIM in their business models. High interest margins are usually explained by high credit losses. In these cases, GI could be an inappropriate proxy for operational risk exposure. To deal with the issue of a high interest margin, Basel II authorised the replacement of GI by an asset-based proxy (loan and advances multiplied by a fixed m-factor of 0.035) in two business lines (retail and commercial banking) under the ASA.

42. The potential for a high or low NIM to result in an inappropriate operational risk capital estimate remains under the revised SA, since the BI retains net interest as one of its components. The calibration takes into account a material number of banks with different business models and in different countries. However, it may not work properly for banks that deviate too much from the average. As a single common set of regulatory coefficients combined with the BI could lead to a disproportionate capital outcome for ASA banks under these conditions, the new QIS exercise will be used to test alternative solutions for this issue.

The approach to addressing the issue of high or low NIMs

43. One option for dealing with the issue of high/low NIMs is to normalise the interest component included in the BI when it is outside a collar. This involves the multiplication of the BI´s interest component by a ratio of an "interest margin cap" or "interest margin floor" to the "actual interest margin" charged by the bank (*Normalisation Ratio*). The approach is summarised below:

Normalised Interest Component = Net Interest Component * Normalisation Ratio

For high interest margin cases, the normalisation ratio may be linear or non-linear

Linear Normalisation Ratio = Interest Margin Cap/Actual Interest Margin *or*

Non-Linear Normalisation Ratio = ln (Interest Margin Cap)/ln (Actual Interest Margin)

44. Under this proposal, the Committee will use new QIS data to investigate the use of an interest margin cap for high NIM banks and whether the normalisation ratio should be linear or not. The use of a non-linear normalisation ratio might be an appropriate approach for allowing smaller adjustments for higher-margin banks, as an acknowledgement that part of the margin may be used to cover operational risk as well as credit risk.

45. There may also be some banks with a very low NIM, which could result in a capital charge for operational risk that is too low for their operational risk profile. A very low net interest margin could reflect various factors such as competition, low credit losses or low operational risks. If the new QIS suggests that this issue is significant, the Committee would consider developing an appropriate solution for such banks by, for example applying a floor to the interest component. Setting a floor, as well as a cap to the NIM, could also make the application automatic for banks with a NIM lying outside the "collar", thus ensuring a consistent application of the adjustment to the interest component. The Committee intends to further explore an appropriate mechanism – automatic versus supervisory judgement – for activating the adjustment.

Other issues

46. A small number of banks that are highly specialised in fee businesses have been identified as facing a disproportionately high capital impact under the BI. The problem stems from the structure of the BI, which was designed to capture the operational risk profile of a universal bank and does not lend itself to accurate application in the case of banks engaged predominantly in fee-based activities. The Committee will respond to the issue if it is evidenced by the results of the new data collection exercise.

> Q8. Do the issues of high interest margin and highly fee specialised businesses in some jurisdictions need special attention by the Committee? What could be other approaches to addressing these issues?

Risk management expectations under the revised SA

47. The Basel framework recognises that capital is not a substitute for effective controls and risk management processes. Rather, strong and effective risk management and internal control processes help reduce the capital that a bank needs to hold against its operational risks. An emphasis on sound management of operational risk to ensure financial soundness of banks is consistent with the uncertainties in the current capital measurement methodologies for operational risk, which are improved but still evolving toward maturation.

48. The Committee's Principles set out its expectations for the management of operational risk. All internationally active banks should implement policies, procedures and practices to manage operational risk commensurate with their size, complexity, activities and risk exposure, and they should seek continuous improvement in these activities as industry practice evolves. Internationally active banks with significant operational risk exposures should be evolving towards the qualitative risk management standards of banks using the AMA for capital estimation, even while they remain on the SA.

49. In the current operational risk regulatory framework, the TSA/ASA has explicit qualifying criteria for risk management. Because the revised SA approach will become the "entry level" capital methodology, its use will not require supervisory approval nor will it be accompanied by any explicit operational risk management standard. This does not mean, however, that the revised framework is rendered less rigorous than the existing one, as this would not be appropriate in the light of the substantial operational risk losses incurred by banks during and in the aftermath of the recent financial crisis. Therefore, the Principles remain applicable to all banks.

50. **Annex 4** provides an example of guidance regarding the qualitative standards that should be observed by large internationally active banks under Pillar 2. The implementation of Principles by larger banks is expected to be more definitive, rigorous and comprehensive. This will also help align supervisory expectations across these banks. Supervisory authorities may go beyond this guidance depending upon the complexity of a bank's operations and its operational risk profile. In addition, supervisory authorities may choose to apply these standards to non-internationally active as well as smaller internationally active banks.

> Q9. What would be the most effective approach to promoting rigorous operational risk management at banks, particularly large banks?

Annex 1: Definition of the Business Indicator

Income statement "Segment" or Macro-Component	Income statement "Item"	Use within the Business Indicator	Description of the "Item"	Typical sub-items
"Interest"	Interest income	Abs (Interest Income - Interest Expenses)	Interest income from all financial assets, both primary financial instruments (included either in trading or non-trading books) and hedge accounting derivatives, as well as other interest income.	Interest income from loans and advances
				Interest income from Available For Sales, Held to Maturity. Fair Value Option, Held for Trading
				Interest income from hedge accounting derivatives
				Other interest income
	Interest expense		Interest expense from all financial liabilities, both primary financial instruments (included either in trading or non-trading books) and hedge accounting derivatives, as well as other interest expenses.	Interest expenses from deposits
				Interest expenses from debt securities issued
				Interest expenses from hedge accounting derivatives
				Other interest expenses
"Services"	Fee and commission income	+	Income received for providing fee-based advices and services referring to both on-balance and off-balance sheet activities. It should also include income received as provider of financial services.	Fee and commission income from:
				- securities (issuance/origination or reception/transmission/execution of orders on behalf of customers)
				- clearing and settlement
				- asset management
				- custody
				- fiduciary transactions
				- payment services
				- structured finance
				- servicing from securitisation activities
				- loan commitments and guarantees given

				- foreign transactions
	Fee and commission expenses	+	Expenses paid for receiving fee-based advices and services referring to both on-balance and off-balance sheet activities. It should also include all expenses paid for outsourced financial services.	Fee and commission expenses for:
				- clearing and settlement
				- custody
				- servicing fees for securitisation activities
				- loan commitments and guarantees received
				- foreign transactions
	Other operating income	+	Income from ordinary banking operations not classified in other BI items but of similar nature.	Rental income from investment properties
				Income from financial leasing and operating leasing
				Gains from non-recurrent assets and disposal group classified as held for sale not qualifying as discontinued operations
	Other operating expenses	+	Expenses and losses from: (i) ordinary banking operations not classified in other BI items but of similar nature (eg fees and commissions, including outsourcing ones), and (ii) operational risk events (not provisioned for in advance).	Expenses for financial leasing and operating leasing
				Losses from non-recurrent assets and disposal group classified as held for sale not qualifying as discontinued operations
				Direct charges to the P&L and costs incurred as a consequence of operational risk events (eg fines, penalties and litigation settlements), which have not been provisioned for in advance
"Financial"	Net Profit (Loss) on financial operations	Abs (Net P&L on TB) + Abs (Net P&L on BB)	Net gains/losses on financial operations (both trading and banking books)	Net gains/losses on financial assets and liabilities held for trading (derivatives, debt securities, equity securities, loans and advances, short positions, other assets and liabilities)
				Net gains/losses on financial assets or liabilities measured at fair value through profit or loss
				Realised net gains/losses on financial assets and liabilities not measured at fair value through profit or loss (available for sale financial assets, loans and advances, held to maturity investments, financial liabilities measured at amortised cost)
				Net gains and losses from hedge accounting
				Net exchange differences

The following sub-items should not contribute to any of the items of the Business Indicator:

- Dividend income
- Income and expenses from insurance or reinsurance business
- Premiums paid and reimbursement/payments received for insurance or reinsurance policies purchased
- Recovery of taxes debited to customers
- Administration expenses: staff expenses (including salaries, pension and similar benefits), outsourcing fees paid for the supply of non-financial services (ie logistical, IT, human resources), other administrative expenses (including expenses for IT, utilities, telephone, travel, office supplies, postage etc)
- Expenses on share capital repayable on demand
- Net gains/losses on derecognition of financial assets, non-financial assets, liabilities not measured at fair value through profit or loss
- Depreciation/amortisation (eg on properties, tangible assets, intangible assets)
- Provisions/reversal of provisions (eg on pensions, commitments and guarantees given, legal issues)
- Impairment/reversal of impairment (eg on financial assets, non-financial assets, investments in subsidiaries, joint ventures and associates)
- Negative goodwill recognised in profit or loss
- Share of the profit or loss of investments in subsidiaries, joint ventures and associates
- Income tax, corporate tax (tax based on profits, including current tax and deferred tax)

Annex 2: The OpCaR methodology

I. Introduction

1. This Annex describes the "OpCaR calculator", ie the methodology developed by the Committee to estimate a bank's operational risk capital-at-risk, or OpCaR. As noted in the main document, the estimates obtained from the OpCaR calculator are used for two purposes: (i) to inform the regressions undertaken to assess the risk sensitivity of the investigated indicators (eg BI, GI, total assets etc); (ii) to calibrate the coefficients to be applied to the proxy indicator selected for the revised SA, ie the BI.

2. The OpCaR calculator was developed and validated based mainly on the operational risk Quantitative Impact Study (the "QIS data"), which was part of the 2010 comprehensive QIS performed by the Committee. More specifically, the QIS exercise gathered information for the period 2005–09 at both the banking group and business line levels on balance sheet and income statement items, and on the number and amount of operational risk losses above specific thresholds. The data elements used in the OpCaR methodology are summarised in Table A.1.

Data elements used in the "OpCaR calculator" Table A.1

Year	Information on operational risk losses			
	Number of loss events ≥ €10,000	Number of loss events ≥ €20,000	Total amount of losses ≥ €20,000	Maximum loss
2005	n_1'	n_1	S_1	M_1
2006	n_2'	n_2	S_2	M_2
2007	n_3'	n_3	S_3	M_3
2008	n_4'	n_4	S_4	M_4
2009	n_5'	n_5	S_5	M_5

3. Section 2 of the Annex describes the main features of the OpCaR calculator. Section 3 illustrates the process adopted for validating the most relevant assumptions of the methodology and Section 4 focuses on the statistical elements and assumptions of the OpCaR calculator.

II. The main features of the OpCaR calculator

The OpCaR model

4. The OpCaR methodology estimates a bank's operational risk capital through the convolution of a single severity distribution and a single frequency distribution. Each bank's OpCaR estimate was assumed to refer to a unique operational risk category, having a specific aggregated frequency and severity of losses.

5. The number of losses between €10,000 and €20,000, the amount of losses (above €20,000) and the maximum loss reported in each reference year were used to represent the bank's whole distribution of loss, from which the percentile at the 99.9 confidence level was determined through the single loss approximation (SLA) formula. The SLA allows high percentiles to be obtained for the aggregated frequency-severity distribution under specific conditions.

6. To obtain a more stable data source for OpCaR estimation, only one figure for the frequency and severity of losses was computed for each bank by averaging the data across the reporting years and, in a specific model, considering the largest value of the "maximum loss" data.

7. It is important to note that the OpCaR methodology does not replicate a typical AMA model as the latter incorporates information relating to scenario analysis and business environment and internal control factors. Information on these two components was not collected in the QIS exercise. The exclusions of these elements may cause, in some cases, an underestimation of the OpCaR figures mainly because scenario analysis data are sometimes included in banks' AMA models to estimate infrequent and large tail events. This potential underestimation was addressed by a more conservative choice of the class of severity distributions in the OpCaR calculator.

8. The QIS data did not allow any fitting of banks' individual loss experiences, either for the frequency or for the severity distributions, because the losses were reported in aggregated form. However, the QIS data were compatible with the methodology often used in scenario-based approaches, which make use of aggregate statistics to compare empirical and theoretical quantities – through "moment" and "percentile" matching estimation methods – for assumed distributions of frequency and severity of losses.

9. Initially, following a scenario-like approach, while the frequency of operational risk losses was assumed to follow a single distribution (the Poisson), six different severity distributions were used covering a wide spectrum of tail behaviour in the data: log normal, log gamma, log logistic, Pareto "light", Pareto "medium" and Pareto "heavy" (see Section 3). The flexibility thus introduced in the OpCaR calculator made it possible to generate up to six OpCaR estimates per bank, each of them referring to a different frequency-severity model of the aggregated loss distribution.

Use of filters to determine appropriateness of the OpCaR model for a bank:

10. Even if, ideally, six models and OpCaR figures per bank can be generated by the calculator, the final outcome depends strictly on a bank's loss experience and structure. In some cases, the algorithm may not find a solution for matching the empirical and theoretical moments or percentiles and this is a clear sign of the inconsistency between the employed OpCaR model and the underlying bank's loss experience. Furthermore, even where the procedure is able to generate an OpCaR figure, this outcome may still not be consistent with a bank's loss experience. In order to detect such situations, specific "acceptance ranges" were identified for key statistical ratios and parameters of each frequency-severity model. These ranges were appositely generated by the calculator in addition to the OpCaR figure.

11. In particular, two filters were initially considered to determine whether each model and the associated OpCaR had to be considered in the bank's capital estimate: (i) whether the proportion of losses above €20,000 was within a certain range (1%–40%); (ii) whether the ratio between loss frequency and total assets was within a certain range (0.1–70 losses per € bn of assets). The ranges considered in the filters were informed by the Loss Data Collection Exercise (LDCE) of 2008. The bounds set were intended to be sufficiently wide to include as many OpCaR estimates as possible in the analysis. Furthermore those outcomes that implied infinite-mean models were also excluded from the calculation, as this would have generated capital figures that did make economic sense. In the end, for each bank, the models that did not find a solution or that produced an OpCaR accompanied by ratios and/or parameters outside the "acceptance ranges" were excluded from the final calculation. If no model fulfilled these conditions, no OpCaR was selected and the bank was excluded from the panel. As

explained in the following section, the filters and acceptance ranges initially set were then validated on the actual data and where necessary were modified.

12. All models that surpassed the filters were included in a bank's final OpCaR estimate. This estimate was obtained by the simple average of the accepted OpCaRs, taking into account the fact that an average across models is a simple and robust way to accommodate possible differences in outcomes.

The validation of the OpCaR methodology

13. The OpCaR calculator was run and validated on a sample of 121 out of 270 QIS banks which were able to provide data on operational risk losses of adequate quality (Table A.2). The sample covered banks following all the capital measurement approaches for operational risk set out under the Basel framework.

14. The objective of validation was two-fold: (a) assessing the ability of each of the six candidate models – each model is a combination of the Poisson distribution and one of the six severity distributions – in producing a usable OpCaR estimate and (b) verifying the power and effectiveness of the set filters and pertinent acceptance ranges.

15. The capacity of an OpCaR model to produce reliable estimates was assessed based on the number of times each of the distributions survived the model filters. Four of the distributions (log normal, log gamma, Pareto medium, and Pareto heavy) were selected for the final OpCaR calculation around 20% of the time or less. The frequent rejection of these candidate distributions is due to the convergence failure of the procedure or to ratios outside the acceptance ranges.

16. For a majority of banks, two distributions, the log logistic and the Pareto light, produced acceptable estimates according to the filters used in the OpCaR methodology. Their higher acceptance rate than the other four distributions is most likely due to the fact that these two distributions locate themselves in the middle of the six candidate models as to tail heaviness, and hence they are more effective in accommodating the bank's loss data behaviour at group level. When comparing the two distributions, the Pareto light model led to a few more unstable results, sometimes explosive, with some regularity, while the log logistic did not, in most cases, produce unreasonable results, even in the face of the sparse data available for OpCaR estimation. Also, the log logistic provided meaningful results when the input data were stressed in some years with large aggregated loss figures.

17. As regards the power and effectiveness of the set filters and pertinent acceptance ranges, it was first observed that relaxing the exclusion criterion based on the proportion of losses above €20,000 did not produce unreasonable results. Thus, the exclusion condition regarding losses above €20,000 was relaxed from excluding banks when their proportion of estimate losses above €20,000 was below 1% or above 40%, to only excluding banks when the said proportion was below 1% or above 80%. As to the ratio between the frequency of reported operational losses and total assets, the validation showed that this filter applies almost always when the €20,000 filters also apply. Therefore, this restriction was eliminated from the selection process.

18. The analytical work indicated the need to include an extra restriction not previously considered. The OpCaR figure for a few banks implied a ratio between unexpected loss (UL) and expected loss (EL) below one. This result was seen as implausible because the OpCaR calculation was done at a bank group

level and not at a specific risk class, where such a ratio might occur. Therefore a new filter requiring the ratio of UL to EL to be above one was introduced.[7]

19. After the validation of the methodology, 99 banks remained with usable OpCaR estimates, corresponding to a survival rate of 82% of the banks in the original QIS sample (Table A.2). In the rest of this Annex and in **Annex 3**, the sample of banks with usable OpCaR estimates is referred to as the "OpCaR data set".

Composition of sample of banks selected by the OpCaR calculator

Table A.2

	Group 1	Group 2	AMA	TSA	ASA	BIA	Total
No. of QIS banks with adequate operational risk data	74 (61%)	47 (39%)	37 (31%)	57 (47%)	4 (3%)	23 (19%)	121
No. of QIS banks selected by the OpCaR methodology (ie "OpCaR data set")	99		29	46	4	20	99
Banks survival rate in the "OpCar data set"	82%		78%	81%	100%	87%	82%

III. The statistical elements and assumptions of the OpCaR calculator

20. The OpCaR calculator is based on the statistical methodology of the LDA, which banks commonly use in their AMA models for calculating operational risk regulatory capital. However, where necessary, further techniques and principles from the field of actuarial science were used.[8]

21. In an LDA context, a bank's aggregate loss distribution is typically represented by a random sum:

$$S = \sum_{i=1}^{N} X_i \qquad (1)$$

Where:

- S is the aggregated loss variable and N and X_i are appropriate mutually independent random variables representing, respectively, the number of events observed in a specific time horizon (the frequency), and the severity of each of those events (the severity);
- N is a discrete random variable assuming non-negative values; the most common choice for the distribution of the frequency in an operational risk context is the Poisson with mean $E[N] = \lambda$;

[7] At the same time, the analytical work verified that the ratio UL/EL for several models did not exceed the theoretical limit implied by the underlying severity distributions.

[8] Useful theoretical references are (i) S Klugman, H Panjer and G Willmot, *Loss Models – From Data to Decisions*, second edition, John Wiley and Sons, New York, 2004; (ii) P Embrechts, C Kluppelberg and T Mikosch, *Modelling Extremal Events for Insurance and Finance*, Springer-Verlag, Berlin, 1997; (iii) R Kreps, *Continuous Distributions, Instrat® Working Paper*, Guy Carpenter & Company Inc, 1998.

- X_i is, for every i, a non-negative, continuous random variable. In the operational risk context, the severity distribution is typically positively skewed and medium to heavy tailed. In statistical terms, this may mean that not all of the statistical moments of the severity distribution exist. As pointed out by the Committee's "Operational risk – supervisory guidelines for the advanced measurement approaches (July 2011)", "*in such cases the use of so-called sub-exponential distributions*[9] *is highly recommended*" (emphasis added; page 40).

22. If N is distributed as a Poisson random variable, and X is a sub-exponential random variable, then it can be shown[10] that a percentile level p, with p close to 1, can be approximated by the following, relatively simple expression (usually referred to as "single loss approximation, or SLA, with mean correction"):

$$F_S^{-1}(p) = F_X^{-1}\left(1 - \frac{1-p}{\lambda}\right) + (\lambda - 1) \cdot E[X] \qquad (2)$$

where F_S^{-1} and F_X^{-1} are the percentile functions of S and X, respectively, and E[X] is the mean of X. In formula (2), the first term represents the unexpected loss, or UL, and the second term represents the expected loss, or EL. Expressions like formula (2) are referred to as percentiles of compound Poisson random variables, which are represented here as $S \sim \text{CPoisson}(\lambda, F_X)$.

23. The application of the standard LDA to the QIS data requires some assumptions to be made. Therefore we define OpCaR = $F_S^{-1}(p)$ with p = 99.9%, and assume that E[X] always exists and that $\lambda = E[N] >> 1$. Moreover, we assume that the frequency is represented by a Poisson distribution and that the severity belongs to the family of sub-exponential distributions. With the use of sub-exponential distributions, we exclude light-tailed distributions for the severity; and by doing so we aim to overcome the potential underestimation of the OpCaR that may result from not including potentially large losses coming from scenario analysis (see the previous section).

24. We consider the sub-exponential distributions for the severity as indicated in Table A.3. These distributions, which are very common in an operational risk context, range from heavy-tailed distributions to medium-tailed distributions.[11] Proper constraints are set over the parameters in order to have well-behaved densities and exclude infinite-mean models.

List of severity distributions used in the OpCaR methodology

Table A.3

Severity distribution (X)	(F_X) of the Severity distribution	Constraints on parameters
1. Pareto(θ,α)	$1-\left(\frac{\theta}{x+\theta}\right)^{\alpha}$	$\theta > 0$, $\alpha > 1$

9 Sub-exponential distributions are those distributions whose tails decay slower than exponential distributions. The class of sub-exponential distributions includes the log normal, log-normal-gamma, log-gamma, log-logistic, generalised Pareto, Burr, Weibull (with shape parameter < 1). The Weibull (with shape parameter > 1) and gamma distributions do not belong to the class of sub-exponential distributions.

10 K Böcker and C Klüppelberg, "Operational VaR: a closed-form approximation", Risk Magazine, vol 18, no 12, 2005, pp 90–3; K Böcker and J Sprittulla, "Operational VaR: meaningful means", Risk Magazine, December 2008, pp 96–8.

11 The severity curves adopted in the exercise cover a wide range of possible different tail behaviours within the medium-heavy tailed domain. The introduction of further curves, while increasing the complexity of the model, most likely would not have provided significant improvements to the estimates.

2. Log normal(μ,σ)[12]	$\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)$	σ > 0
3. Log logistic(θ,α)	$\left[1+\left(\frac{\theta}{x}\right)^{\alpha}\right]^{-1}$	θ > 0, α > 1
4. Log gamma(α,τ)[13]	$G_{\tau}(\alpha \cdot \ln(x))$	τ > 0, α > 1

25. The EL, UL and OpCaR for compound Poisson variables with the listed severity distributions may be analytically estimated using the SLA according to the formula given by (2). Table A.4 shows the EL and UL for several compound Poisson models; OpCaR is calculated as the sum of EL and UL.

UL and EL expressions based on the SLA for the selected compound Poisson models

Table A.4

Compound Poisson Model (λ,F_X)	Expected Loss (EL)	Unexpected Loss (UL)
1. CPoisson(λ,$F_{Pareto(\theta,\alpha)}$)	$\theta \cdot \frac{\lambda-1}{\alpha-1}$	$\theta \cdot \left[\left(\frac{1-\pi}{\lambda}\right)^{-1/\alpha}-1\right]$
2. CPoisson(λ,$F_{log\ normal(\mu,\sigma)}$)	$(\lambda-1)\cdot \exp\left\{\mu+\frac{\sigma^2}{2}\right\}$	$\exp\left\{\mu+\sigma\cdot\Phi^{-1}\left(1-\frac{1-\pi}{\lambda}\right)\right\}$
3. CPoisson(λ,$F_{log\ logistic(\theta,\alpha)}$)	$\theta\cdot(\lambda-1)\cdot\Gamma\left(1+\frac{1}{\alpha}\right)\cdot\Gamma\left(1-\frac{1}{\alpha}\right)$	$\theta\cdot\left[\left(1-\frac{1-\pi}{\lambda}\right)^{-1}-1\right]^{-1/\alpha}$
4. CPoisson(λ,$F_{log\ gamma(\alpha,\tau)}$)	$(\lambda-1)\cdot\left(1-\frac{1}{\alpha}\right)^{-\tau}$	$\exp\left\{\frac{1}{\alpha}G_{\tau}\left(1-\frac{1-\pi}{\lambda}\right)\right\}$

26. For a compound Poisson model (λ,F_X), it is possible to demonstrate that if a threshold u > 0 belongs to the X domain,[14] then the Frequency of the losses above the threshold, λ_u, is

$$\lambda_u = E[N \mid X \geq u] = \lambda \cdot [1 - F_X(u)] \tag{3}$$

27. Therefore, the estimate of the Poisson parameter may be obtained by:

$$\lambda = \lambda_u \cdot [1 - F_X(u)]^{-1} \tag{4}$$

12 $\Phi(z)=\frac{1}{\sqrt{2\cdot\pi}}\int_{-\infty}^{z} e^{-\frac{x^2}{2}}dx$ is the *standard normal distribution*; the Φ's domain is (-∞, +∞).

13 $G_{\alpha}(z)=\frac{1}{\Gamma(\alpha)}\int_{0}^{z} x^{\alpha-1}\cdot e^{-x}dx$ is the *standard gamma distribution*. The G's domain is [0, +∞) and the shape parameter α must be greater than 0. $\Gamma(\alpha) = \int_0^{+\infty} x^{\alpha-1}\cdot e^{-x}\cdot dx$, with α > 0, is called *gamma function*.

14 We mean the set of possible values for X.

28. In the QIS data, there are two different thresholds: u = €20,000 and u′ = €10,000. We can, therefore, get two different frequency figures, λ_u and $\lambda_{u'}$, by averaging the number of losses above those thresholds that occurred in the QIS reporting years:

$$\lambda_u = \frac{\sum_{j \in T} n_j}{|T|}, \text{ and}$$

$$\lambda_{u'} = \frac{\sum_{j \in T} n_j'}{|T|} \tag{5}$$

where T = set of reporting years and "| … |" stands for number of elements.

29. The estimate of the Poisson parameter, λ, in the formula given by (4) requires that the parameters of the severity distributions are estimated first. Because all of the assumed severity distributions have two-parameters, we need to set an equation with at least two conditions for the estimation of the parameters.

30. For this purpose, we note that we can compute the empirical mean of X above the threshold u = €20,000 (ie the empirical "conditional mean" above u) from the QIS data as:

$$E[X \mid X \geq u] = \frac{\sum_{j \in T} s_j}{\sum_{j \in T} n_j} \tag{6}$$

31. Therefore, we can get the first condition, a *moment condition,* by deriving the theoretical "conditional mean" (above u) for the selected severity distributions and matching these values with the empirical ones. This condition is sensitive to the bulk of the aggregated losses.

Theoretical "conditional mean" (above u) for the selected severity distributions — Table A.5

Severity distribution (X)	Conditional mean E[X | X > u]
1. Pareto(θ,α)	$u + \frac{\theta + u}{\alpha - 1}$
2. Log normal(μ,σ)	$e^{\mu + \frac{\sigma^2}{2}} \cdot \Phi\left(\frac{\mu + \sigma^2 - \ln(u)}{\sigma}\right) \cdot \left[\Phi\left(\frac{\mu - \ln(u)}{\sigma}\right)\right]^{-1}$
3. Log logistic(θ,α)[15]	$\frac{\theta \cdot \Gamma(1 + 1/\alpha) \cdot \Gamma(1 - 1/\alpha) \cdot [1 - B_{1+1/\alpha, 1-1/\alpha}(1/[1 + (\theta/u)^\alpha])]}{1 - 1/[1 + (\theta/u)^\alpha]}$
4. Log gamma(α,τ)	$\frac{(1 - 1/\alpha)^{-\tau} \cdot [1 - G_\tau((\alpha - 1) \cdot \ln(u))]}{1 - G_\tau(\alpha \cdot \ln(u))}$

15 $B_{\alpha,\beta}(z) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha) \cdot \Gamma(\beta)} \int_0^z x^{\alpha-1} \cdot (1-x)^{\beta-1} dx$ is the *standard beta distribution*. The $B_{\alpha,\beta}$'s domain is [0,1] and the shape parameters α and β must be greater than 0.

32. A second condition may be derived by observing that the ratio $\lambda_{u'}/\lambda_u$, according to (4), can be expressed as the following (threshold-driven) "percentile ratio":

$$\lambda_{u'}/\lambda_u = [1 - F_X(u')] / [1 - F_X(u)] \qquad (7)$$

33. From (5) an empirical estimate of $\lambda_{u'}/\lambda_u$ may be obtained through n_j and n_j'. Therefore, we can get the second condition, a *percentile condition,* by deriving the theoretical (threshold-driven) percentile ratios and matching them with the empirical ones. This condition is sensitive to the region of small losses (those between €10,000 and €20,000).

Theoretical (threshold-driven) "percentile ratios" for the selected severity distributions

Table A.6

Severity distribution (X)	Threshold-driven "Percentile ratio" $[1 - F_X(u')] / [1 - F_X(u)]$
1. Pareto(θ,α)	$\left(\frac{u+\theta}{u'+\theta}\right)^{\alpha}$
2. Log normal(μ,σ)	$\Phi\left(\frac{\mu-\ln(u')}{\sigma}\right)\cdot\left[\Phi\left(\frac{\mu-\ln(u)}{\sigma}\right)\right]^{-1}$
3. Log logistic(θ,α)	$\frac{1-[1+(\theta/u')^{\alpha}]^{-1}}{1-[1+(\theta/u)^{\alpha}]^{-1}}$
4. Log gamma(α,τ)	$\frac{1-G_{\tau}(\alpha\cdot\ln(u'))}{1-G_{\tau}(\alpha\cdot\ln(u))}$

34. In addition to the "moment" and the "percentile" conditions, we can derive another approximate relation by looking at the behaviour of the severity distributions around their maxima.

35. If $X_{(n)} = \text{Max}\{X_1, X_2, \ldots, X_n\}$, then we have, in general, that:

$$E[F_X(X_{(n)})] = n\cdot(n+1)^{-1} \qquad (8)$$

36. Therefore, when $n \to +\infty$, we can expect that $E[F_X(X_{(n)})] \approx F_X(E[X_{(n)}]) \approx F_X(M)$, where M is a suitable estimation of $E[X_{(n)}]$.

37. In the case of a Pareto distribution (conditional above the threshold u'), (8) becomes:

$$E[F(X_{(n)})\,|\,X \geq u'] \approx F_{X|X\geq u'}(M) = \left[\left(\frac{\theta}{u'+\theta}\right)^{\alpha} - \left(\frac{\theta}{M+\theta}\right)^{\alpha}\right]\cdot\left(\frac{\theta}{u'+\theta}\right)^{-\alpha} \approx \frac{n'}{n'+1} \qquad (9)$$

38. The formula that is given by (9) can be seen as a "*maximum condition*", that is a condition that is sensitive to the region of very large losses.

39. Therefore, from the QIS data, we can estimate M in two ways, either as the largest or as the average of the "Maximum Loss" data, which can be expressed as:

$$M = Max_{j=1..t}\{M_j\} \text{ or, } M = \frac{\sum_{j=1}^{t} M_j}{t} \qquad (10)$$

40. Obviously, in the first case we suppose a heavier tail than in the second case. Therefore, we can refer to the "maximum condition" by splitting it between the "maximum-heavy condition" and the "maximum-medium condition".

41. In the case of the Pareto distribution, up to six reference models can be obtained depending on which two of the four conditions described above are used (moment, percentile, maximum-heavy, maximum-medium) for the estimate of the parameters. We decided to limit the Pareto to three models, keeping the moment condition as the first condition in all three and using the percentile condition as the second condition in one case and the two variations of the maximum conditions as the second condition in the other case.

42. Therefore, the number of initial four compound Poisson models reported in Table A.7 is expanded to six after the inclusion of the two additional Pareto variations.

Compound Poisson models and pertinent conditions for the severity parameters estimate

Table A.7

Compound Poisson Model (λ,F_X)	1st condition on the F	2nd condition on the F
1a. CPoisson(λ,$F_{Pareto(\theta,\alpha)}$) – light	Moment (bulk)	Percentile (small losses region)
1b. CPoisson(λ,$F_{Pareto(\theta,\alpha)}$ - medium)	Moment (bulk)	Maximum-medium (large losses region)
1c. CPoisson(λ,$F_{Pareto(\theta,\alpha)}$ - heavy)	Moment (bulk)	Maximum-heavy (large losses region)
2. CPoisson(λ,$F_{log\ normal(\mu,\sigma)}$)	Moment (bulk)	Percentile (small losses region)
3. CPoisson(λ,$F_{log\ logistic(\theta,\alpha)}$)	Moment (bulk)	Percentile (small losses region)
4. CPoisson(λ,$F_{log\ gamma(\alpha,\tau)}$)	Moment (bulk)	Percentile (small losses region)

43. In each model, the two conditions for the parameters estimate of the severity distribution need to be solved[16] by an iterative procedure. In the case of the log normal (ie CPoisson (λ,$F_{log\ normal(\mu,\sigma)}$)), the starting points of the iterative procedure were obtained from the 2008 LDCE data. For all of the other models, the starting points were based on the indications provided in the literature.[17]

Starting point for the parameters estimate of the severity distribution of the compound models

Table A.8

Compound Model (λ,F_X)	θ	α	μ	σ	τ
1. CPoisson(λ,$F_{Pareto(\theta,\alpha)}$)	$E[X \mid X \geq u]$	2	-	-	-
2. CPoisson(λ,$F_{log\ normal(\mu,\sigma)}$)	-	-	10	1	-
3. CPoisson(λ,$F_{log\ logistic(\theta,\alpha)}$)	$E[X \mid X \geq u]$	2	-	-	-
4. CPoisson(λ,$F_{log\ gamma(\alpha,\tau)}$)	-	2	-	-	2

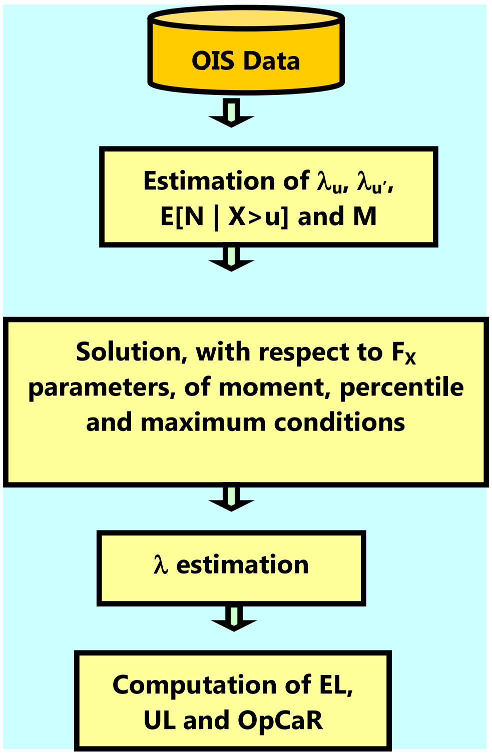
44. From the estimate of the parameters of the severity distributions, the estimate of the λ parameter of the Poisson distribution may be derived by (4). Finally the EL, UL and OpCaR can be computed by replacing the frequency and severity parameter estimates in the SLA expressions in Table A.4.

16 For practical implementation see Klugman et al, 2004, pp 659–69 (Appendix F).

17 See in particular, Klugman et al, 2004, pp 630–31.

45. The logical sequence of the described steps to calculate OpCaR is summarised in the following workflow diagram.

Figure A.1: Workflow of the OpCaR methodology



46. It is important to note that, for each bank, where the iterative procedure of a compound model was able to generate an estimate of the parameters for the severity distribution, the pertinent OpCaR was derived. All of the models that generated OpCaR figures entered into the filtering process described in the previous sections to arrive at the bank's final OpCaR as an average of the OpCaR figures of the surviving models.[18] Conversely, where the iterative procedure did not converge for a severity distribution, the model was excluded from the following calculation steps.

[18] Given the low dispersion and the small number of OpCaRs selected per bank, model averaging was preferred over choosing the maximum or the minimum OpCaR figure. The adoption of a different criterion (ie median, maximum, minimum) would have resulted in substantial differences in very few cases.

Annex 3: Investigation of 20 potential indicators of operational risk exposure

I. Introduction

1. This Annex describes the analysis undertaken by the Committee to investigate the sensitivity to operational risk of several proxy indicators built over balance sheet and income statement items.

2. As operational risk pervades the range of bank business activities, it stands to reason that proxy indicators that capture relevant balance sheet and income statement items should reflect the range of potential operational risks. However, in order for an indicator to be an effective proxy of operational risk for regulatory purposes, it is crucial that specific banking characteristics are captured by its components.

3. The explanatory power or sensitivity to operational risk of an indicator (that is to say its ability to capture a bank's operational risk exposure) is the most important property for the purposes of this analysis. An indicator that is not sufficiently risk-sensitive implies there is little relationship between the level of the indicator and the operational risk it is being used to represent. An indicator with only limited explanatory power would result in systematically biased regulatory figures, with distortions that may assume any direction (ie positive or negative) and size. Therefore, a poorly designed indicator may require banks to hold too much or too little capital.

4. On the basis of the data and information available in the BCBS 2010 QIS, more than 20 proxy indicators built from balance sheets and/or income statement items were investigated. Some of them were significantly different from the current regulatory proxy, Gross Income, or GI (eg total assets, provisions, administrative costs) others were a refinement of the GI itself. "Refinement" means that these indicators included most or all the items of the GI and the difference was only in the way these items were treated within the proxy (ie added or taken in absolute terms rather than subtracted).

5. In addition to analysing the relevance of the 20 candidate indicators, the analysis also evaluated whether the successful candidate indicators demonstrated a linear or a non-linear relationship with OpCaR. The accuracy of the model implied by each indicator was measured by appropriate goodness-of-fit statistics.

6. The analysis revealed that linear models are inadequate to express such a relationship. This implies that the output of accuracy measures commonly adopted in a linear context (ie R^2 or adjusted-R^2) is unreliable and meaningfulness in detecting and comparing the explanatory power of the investigated indicators. More powerful alternative performance measures – the so-called Akaike Information Criteria (AIC) and Bayesian Information Criteria statistics (BIC) – were therefore investigated and introduced into the analysis.

7. On the contrary, non-linear relationships more appropriately reflect the link between the OpCaR and some of the selected indicators in the proxy. Intuitively, operational losses mainly result from interactions between risks and banks' operational elements (eg business processes, human behaviour, system features). These elements are dynamic and non-linear in nature and tend to generate disproportionately higher risks and losses as banking organisations become bigger and more complex.

8. Among the combinations of indicators tested as proxies showing a non-linear link with the OpCaR, the BI was overwhelmingly superior in capturing a bank's operational risk exposure.

9. Section 2 described the process of selection of the proxy indictors. Sections 3 to 5 are devoted to estimation of the explanatory power of the indicators using appropriate regression studies. The

appendix to this annex outlines the pros and cons of the relevant proxy indicators investigated in the analysis.

II. The selection of the candidate proxy indicators

10. The indicators selected for investigation comprised alternative variables to GI, refinement of the GI, or components and combination of them, based on the criteria described in the main document.

11. Alternative variables to the GI included balance sheet or income statement items that are significant changes in the indicator components of the proxy compared to the GI, such as total assets, total provisions, administrative expenses.

12. A refinement of the GI means a proxy that includes some or all the indicators that comprise the GI, with the difference being the way these indicators are treated within the proxy. Positive and negative components such as interest income and expenses, fee and commission income and expenses, gains/losses on financial operations, dividend income, other operating income and expenses are added, subtracted or taken in absolute terms with the objective to study their aggregated behaviour with respect to operational risk. By construction, the BI is a GI-refined indicator; it is so labelled because, differently from the GI which measures a bank's profitability, this indicator provides a sense for the level of business activity associated with the bank's profit.

13. By considering the data and information available in the QIS, more than 20 proxy indicators were created and investigated in the analysis. In the case of the use of total assets as the dependent variable, either alone or in combination with other variables, the natural logarithm of the values was also considered.

III. Evaluation of the explanatory power of the proxy indicators

14. The explanatory power of the proxy indicators was evaluated by employing bivariate[19] regression analyses, with the bank's OpCaR being a dependent variable obtained as described in Annex 1. Because just one observation per bank was used as the dependent variable,[20] only one observation for each of the independent variables (ie the proxy indicator) was also considered. This was done by taking the average of the proxy indicator figures over the QIS reporting years.

15. In order to get a more robust and consistent sample for regression purposes, outlier observations were excluded by the data set obtained with the OpCaR calculator (ie the "OpCaR data set" as described in Annex 2,) to mitigate the risk that the regression estimates were affected by very few large points (mainly representing AMA banks) rather than by the bulk of the data, which typically identify BIA or TSA/ASA banks.

19 The bivariate regressions were used to increase the degrees of freedom in the model, besides the purposes of reducing the computational costs and getting a less complicated outcome under a regulatory perspective (ie a set of proxy indicators with different regulatory coefficients is not easy to interpret and implement).

20 It may be useful here to note that the OpCaRs used in the QIS were built over the bank's loss experience as measured by the average number and amount of losses in the whole reporting period (2005–09).

16. The identification of outliers was done using the OpCaR and the BI as the key elements. More specifically, the banks with an extremely large value for BI (BI > €30 bn) or very large ratio with OpCaR (CaR/BI > 50%) were removed by the sample. At the end of this process 10 banks were excluded from the “OpCaR data set”, and the “regression data set” remained with 89 observations. It is important to note that in the “regression data set”, the percentage of non-AMA banks is increased and represents about three quarters of the banks in the sample.

Composition of sample of banks used for regression analyses

Table A. 9

	Group 1	Group 2	AMA	TSA	ASA	BIA	Total
No. of QIS banks with adequate operational risk data	74 (61%)	47 (39%)	37 (31%)	57 (47%)	4 (3%)	23 (19%)	121
No. of QIS banks selected by the OpCaR methodology (ie "OpCaR data set")	99		29	46	4	20	99
Bank survival rate in the "OpCar data set"	82%		78%	81%	100%	87%	82%
No. of QIS banks selected for regression analyses (ie "regression data set")	89		24	41	4	20	89
Bank survival rate in the "regression data set"	74%		65%	72%	100%	87%	74%

1. The explanatory power of the proxy indicators in a linear regression model

17. The first regression analysis assumes a linear relationship between the dependent variable and the proxy indicators. A simplification of the model is further introduced by considering a linear model without intercept:

$$Y = bX + \varepsilon \qquad (1)$$

which in our terminology, becomes:

$$OpCaR_i = b_1 PI_i + \varepsilon_i \qquad (2)$$

where $OpCaR_i$ is the dependent variables, PI_i the independent variable or proxy indicator, b_1 is the parameter to estimate and ε_i the error term, introduced to capture all other factors that influence the y_i other than x_i.

18. The parameter estimate is obtained by an OLS approach, which minimises the following quantities:

$$\sum_i [y_i - b_1 x_i]^2 = \sum_i [\mathrm{OpCaR}_i - b_1 \mathrm{PI}_i]^2 = \min \qquad (3)$$

19. In linear regression, the most common measure of a model’s accuracy, which in our case is the explanatory power of the proxy indicators, is R^2. This measure, which is intuitive, takes on a value between 0 and 1, and is interpreted as the total variance of the dependent variable that is explained by the assumed model. The larger the value of R^2, the better the model fits. The mathematical expression for R^2 is given by (4) below.

$$R^2 = 1 - \frac{RSS}{TSS} \qquad (4)$$

where RSS is the residual sum-of-squares, ($\Sigma\varepsilon_i^2$ or $\Sigma\upsilon_i^2$), and TSS is the total sum-of-squares (ie total variance of the Y_i 's).

20. In applications, the adjusted-R^2, which we denote R^2_{adj} is normally used instead of R^2 to compensate for possible bias due to a different number of observations for the chosen proxy indicators:

$$R^2_{adj} = 1 - \frac{n-1}{n-k} \cdot \left(1 - R^2\right), \qquad (5)$$

where n is sample size and k is the number of parameters, which in our case is 1.

21. However, one aspect that needs to be evaluated before considering the output of the regressions, especially the adjusted-R^2, is whether the assumed linear relationship is appropriate for the variables under investigation. A misspecification of the model may introduce biases in the regression estimates and accuracy measures, including the R^2 or adjusted-R^2, therefore becoming unreliable and may not be useful for comparison of the explanatory power of the indicators.

22. One way to determine whether the model is correctly specified is to graphically review the residuals, ε_i. If the linear model is correct, the residuals, when plotted, appear without any discernable pattern. Therefore, if the residuals appear to behave randomly, then this is an indication that the model fits the data well. On the other hand, if a non-random structure is evident in the residuals, then this is an indication of poor model fit; and therefore alternative models should be investigated.

23. The linear regression on the regression data set was run for all the investigated proxy indicators and the residuals ε_I were obtained and plotted. The residuals for the BI and GI did not exhibit a random structure. In particular, negative residuals appeared more abundant in both the proxy indicators and, in the case of GI, they tended to follow a linear decreasing pattern. Similar results were obtained for all the other variables included in the analysis.

24. In order to confirm (on an analytical basis) the lack of independence of the residuals, a normality test was performed on the residuals for the selected proxy indicators. This was done according to the procedure described in Section 5.2 below. In short, the distance of the empirical distribution of the residuals from a normal distribution N $(0, \sigma^2)$ was measured on the basis of the P-value of a Kolmogorov-Smirnov test. Only the indicators with P-values larger than the significance level of 10% could be considered linearly related to banks OpCaR.

25. For all the selected indicators, the P-values were 0 or under the significance level, and this confirmed the inadequacy of the linear model and R^2-based statistics, respectively, to express and measure the relationship between a bank's op risk OpCaR and the proxy indicators. The investigation of non-linear relationships and alternative accuracy measures was therefore called for.

2. The explanatory power of the proxy indicators in a non-linear regression model

26. In this section, a non-linear relationship between the OpCaR and the proxy indicators is introduced and the explanatory power of the candidate variables is measured.

27. Non-linear models are frequently fitted to data in many fields of applied statistics. A plethora of non-linear models exist, and choosing the right model for the data at hand is a mixture of experience, knowledge about the underlying process and statistical interpretation of the fitting outcome. In order to minimise "model risk", ie the risk of introducing an inappropriate relationship for the proxy indicators under investigation, we opted for a very flexible approach.

28. This approach resorts to a family of functions, summarised in a generalised model that converges to well-known functions when its parameters assume specific values. The generalised model has to be suitable in the sense that it must satisfy economic, statistical and mathematical conditions. From the economic point of view, we expect the function will always be non-negative and monotonically non-decreasing with the proxy indicators. In addition, we looked for a function with slightly increasing average impact.[21] Finally, for practical reasons, we looked for a generalised function sufficiently smooth to allow or simplify mathematical manipulations (in our context sufficiently smooth means twice differentiable). The features and conceptual elements of the approach used in non-linear regressions are described in Section 5.1.

29. We also needed to quantify the model accuracy by some measures that discriminated a "good" fit from a "bad" fit. While R^2 was appropriate for this purpose in the linear regression framework, it is well known that R^2 is an inadequate and inappropriate measure for non-linear regression. This happens because the total sum of squares is not equal to the regression sum of squares plus the residual sum of squares (RSS), as is the case in linear regression, and it therefore lacks the usual interpretation under the linear regime. Application of R^2 to non-linear models generally leads to a measure that can lie outside of the [0,1] interval and decrease as independent variables are added.

30. In order to overcome this problem, alternatives to the R^2 goodness-of-fit statistics were investigated and introduced in the analyses, with a view to providing a much clearer picture of any improved performance, in terms of explanatory power of the proxy indicators and comparison of the *weight of evidence* of one proxy indicator over the others. The features and conceptual aspects of the measures of performance used in non-linear regressions are described in Section 5.2. In Section 5.3 the results of the analysis are reported and discussed.

3. The theory for non-linear regressions: the taxation-like system

31. The principles and criteria over which a progressive taxation system is typically built can be used for the purpose of identifying and estimating non-linear relationships between banks' OpCaRs and the selected proxy indicators.

32. In a typical taxation system, the F(x) (or F) represents the function that provides the coefficients (ie the "average tax") associated with any level of x (ie the "Income"). In this approach, the "total tax" to be paid at a given x is simply the product between x (the income amount) and F (the average tax). If we apply this approach to our case, the operational risk capital requirement that corresponds to a given value of the chosen indicator (ie the proxy indicator) can be expressed as:

$$R(x) = x \cdot F(x), \tag{6}$$

where x is the value of the proxy indicator, F is the coefficient (ie the average requirement) function, and R is the total operational risk requirement.

33. If F is strictly continuous and at least twice differentiable, then it is possible to calculate, among others things, the two derivatives as shown in (7).

$$\begin{aligned} &\text{1st Derivative} = R'(x) = F(x) + x \cdot F'(x), \text{ and} \\ &\text{2nd Derivative} = R''(x) = 2 \cdot F'(x) + x \cdot F''(x), \end{aligned} \tag{7}$$

21 This condition serves to avoid excessively penalising a bank as its size increases. We required that the average impact increases at a decreasing rate, as clarified in Section 5.1 below. Such a condition may induce an increasing marginal impact on the total capital requirement.

where R′, R′′ and F′, F′′ are the first and second derivatives of R and F, respectively. In particular R′(x) identifies the additional capital requirement at a level x for an additional unit of x (ie the marginal requirement).

34. Given that the OpCaR, as computed in Annex 1, represents an estimate of a bank's operational risk exposure, it is logical to associate this with the operational risk capital requirement R(x). Therefore, the inferential problem becomes finding (the best) F and estimating the related parameters that make this relationship work.

35. If F is known and depends on an n-dimensional parameter, say η, the solution may be found in a similar fashion to the approach that is used in Section 4; that is, by applying an OLS estimation with respect to η:

$$\sum_i [\mathrm{OpCaR}_i - x_i \cdot F(x_i;\eta)]^2 = \min \tag{8}$$

36. The formula that is given in (8) is a generalised expression that also encompasses the linear model as a special case where F is a constant. However, the linear model case (that is, where F is a constant) is not considered in this section since it was extensively examined in Section 4.

37. Ideally, an infinite number of F may be tested by solving the minimisation problem in (8). However, the idea here is to look at those F with clear economic meaning, consistent with the F typically adopted in a taxation-like system and whose properties are: (a) F positive and increasing at a constant or decreasing rates; and (b) R′ always above F.

38. It can be shown that if $F(x) > 0$, $F'(x) > 0$ and $F''(x) \le 0$ (condition a), then $R(x) \ge 0$ and $R'(x) \ge F(x)$ (condition b).

39. A well-behaved F(x) that satisfies these conditions, and is commonly used in a taxation-like system, is given in (9),

$$F(x) = \theta \frac{(x - A)^{1-\alpha}}{1 - \alpha} \tag{9}$$

with parameters $A \le 0$, $\theta \ge 0$ and $\alpha \in [0,1]$[22]

40. A useful property of (9) is that it is a very general function, which subsumes a family of elementary functions (eg linear, power and approximately logarithmic), that are obtainable by the appropriate choice of its parameters A, θ or α.[23]

[22] The reasons for the particular shape of the F(·) proposed in the analysis (positivity, monotonic increment with respect to the argument, concavity) stem from the approach adopted in taxation systems. In a typical taxation system, the average "gross tax" is increasing, at a decreasing rate, if and only if the "income" increases. In our framework, the proxy indicator (PI) is the "income" and the OpCaR is the "gross tax"; the difference PI – OpCaR = PI – F(PI). PI is the equivalent of the "net income" after the taxation (or disposable income).

[23] Parameters and analytical expressions of the elementary functions subsumed by the F():

A	θ	α	F(x)	Type
k	q	0	$- q \times k + q \times x$	Linear
-k/θ	ε	0	$\sim k$	Nearly constant (degenerate linear)
-exp{M/θ}	ε × (1 - α)	a	$\sim \varepsilon \times (1 - a) \times \ln(x + \exp\{M/[\varepsilon \times (1 - a)]\})$	Nearly logarithmic
0	q	a	$q \cdot (1 - a)^{-1} \times x^{1-a}$	Power

Where: $\varepsilon = 0+ > 0$; $M >> 0$.

41. From (9), it is easy to see that R and R′ are as follows:

$$R(x) = \theta \frac{x \cdot (x - A)^{1-\alpha}}{1 - \alpha}, \quad (10)$$
$$R'(x) = \theta \frac{(2 - \alpha) \cdot x - A}{(1 - \alpha) \cdot (x - A)^{\alpha}}$$

42. Substituting for (9) in (8), the pertinent OLS minimisation problem is expressed in (11).

$$\sum_i \left[\mathrm{OpCaR}_i - \theta \cdot \frac{x_i \cdot (x_i - A)^{1-\alpha}}{1 - \alpha} \right]^2 = \min, \quad (11)$$

where $A \leq 0$; $\theta \geq 0$; $0 \leq \alpha \leq 1$.

43. The formula given in (9) and its OLS derivation in (11) were applied to the regression data set to estimate a non-linear relationship between a bank's OpCaR and the selected proxy indicators. An iterative procedure was applied to get the parameter estimates. To overcome the risk of non-convergence of the procedure (caused by reaching the parameter bounds during the iterations) the basic formulation was transformed in an equivalent unconstrained one by using ancillary variables. Traditional choices in this sense are:

$$A \equiv - e^{\xi_1},\ \theta \equiv e^{\xi_2},\ \alpha \equiv e^{\xi_3}/(1+ e^{\xi_3}),$$

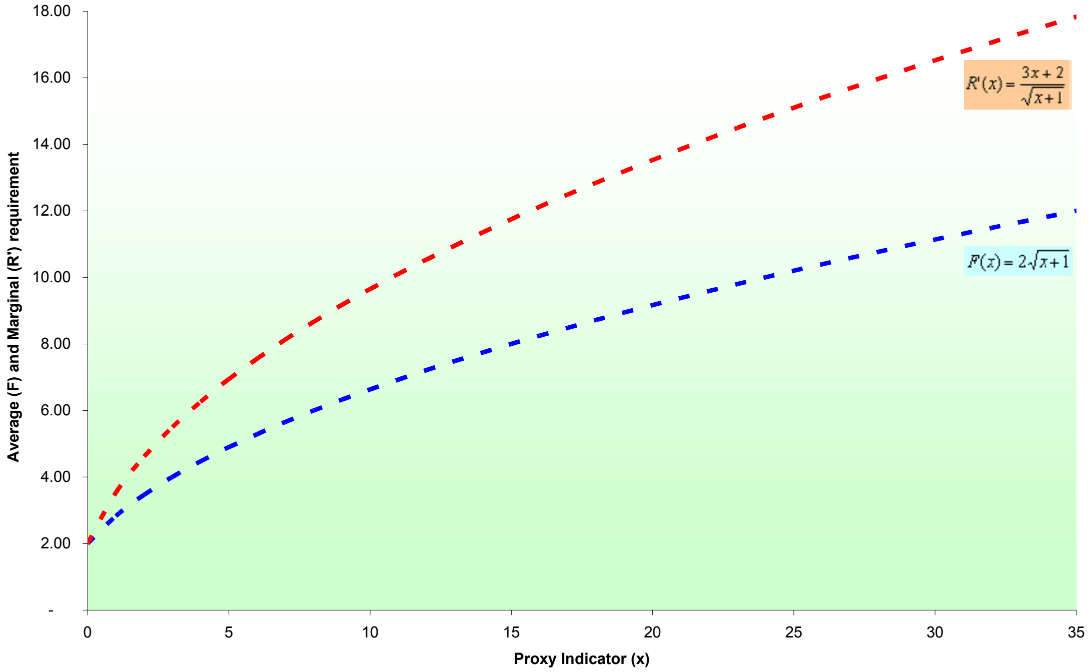
where ξ_1, ξ_2 and ξ_3 assume values on the real line. The equivalent unconstrained OLS minimisation problem therefore became:

$$\sum_i \left[\mathrm{OpCaR}_i - e^{\xi_2} \cdot (1 + e^{\xi_3}) \cdot x_i \cdot (x_i + e^{\xi_1})^{1/(1+e^{\xi_3})} \right]^2 = \min \quad (12)$$

44. A natural choice for the starting point of the optimisation procedure for the unconstrained problem is $\xi_1 = \xi_2 = \xi_3 = 0$, which implies $A = -1$, $\theta = 1$ and $\alpha = ½$. Consequently, starting functions for the average requirement F and marginal requirement R′ are as shown in (13).

$$F(x) = 2\sqrt{x + 1}, \quad (13)$$
$$R'(x) = \frac{3 \cdot x + 2}{\sqrt{x + 1}}$$

Figure A.2: Plot of the average (F) and marginal (R') requirements functions



4. Goodness-of-fit measures for non-linear regressions

45. As R^2–based statistics cannot be used in a reliable way in non-linear regressions, we investigated other goodness-of-fit measures that work in a non-linear framework.

46. Recently, in statistics and other applied sciences, two performance measures have been affirmed for model comparisons. They are: (a) an information theoretic measure that is known as the Akaike Information Criterion (AIC), and (b) a Bayesian measure that is known as the Bayesian Information Criterion (BIC). These goodness-of-fit measures are well suited to our case.

47. The AIC was introduced by Hirotsugu Akaike in his seminal 1973 paper, "Information theory and an extension of the maximum likelihood principle".[24] Akaike extended the traditional maximum likelihood paradigm by considering a framework in which the model dimension is also unknown and must, therefore, be determined from the data. Thus Akaike proposed a framework in which both model estimation and model selection can be simultaneously accomplished.[25]

48. The general expression for the AIC is given in (14).

$$AIC_i = -2 \ln (\mathcal{L}(\eta|\text{data})) + 2k, \tag{14}$$

where η is the maximum likelihood estimation of model parameters, k is the number of parameters, $\mathcal{L}(\cdot|\text{data})$ is the likelihood of the function, ln is the natural logarithm, and i is the i^{th} model (or PI) for which

[24] B Petrov and F Csaki (eds), Second International Symposium on Information Theory, Akademia Kiado, Budapest, pp 267 (281).

[25] The traditional maximum likelihood paradigm, as applied to statistical modelling, provides a mechanism for estimating the unknown parameters of a model having a specified dimension and structure.

the AIC quantity is computed. The term "- 2 ln $\mathcal{L}(\cdot\,|\text{data})$" is called the "goodness-of-fit" term and decreases as the fit of the model improves. The term "2k" is called the "penalty" term and increases as the complexity of the model grows.

49. The main advantage of using the AIC is that this criterion does not require the assumption that one of the candidate models is the "true" or "correct" model. Moreover, the AIC can be used to compare models based on different probability distributions.

50. The corrected version of the AIC, AICc, as shown in (15), is usually adopted in applications in which the number of parameters, k, is large relative to sample size, n (which includes when n is small for any k):[26]

$$\text{AICc} = -2\ln(\mathcal{L}(\eta|\text{data})) + 2k + 2\cdot k\cdot(k+1)/(n-k-1) \tag{15}$$

51. The use of the AIC (or AICc) requires knowing the log-likelihood associated with any candidate model, which in turn requires application of a maximum likelihood estimation (MLE) procedure. In order to use an MLE, one has to assume the type of underlying distribution in order for the appropriate likelihood function $\mathcal{L}(\eta|\text{data})$ to be derived and the parameters estimated.

52. In the non-linear regression described in the previous section, no a-priori distributional assumption is envisaged for F(x), since, as noted earlier, this function actually represents a family of functions. Therefore, this could make it difficult to use an AIC (or AICc) measure for comparing the performance of different proxy indicators. However, it is important to remember that OLS was the procedure used to estimate F(x) and its parameters. One property of OLS estimates is that they are also MLE estimates when the residuals are normally (or Gaussian) distributed. Therefore, where the residuals of the OLS applied to a given model i (ie PI_i) were found to be Gaussian, the OLS equals the MLE, and the AIC (or AICc) measure can be correctly computed.

53. AIC is derived as an asymptotically unbiased estimator of a function used for ranking candidate models based on a divergence measure[27] between the true model and the candidate models. The lower the AIC_i, the better the i^{th} model.

54. Assuming a set of a priori candidate models, then the AIC (or AICc) measure is computed for each of them. The individual AIC_i values are not interpretable in isolation because they contain arbitrary constants and are affected by sample size. Therefore, it is important to rescale the AIC to the best AIC_i value, which as noted previously, is the i^{th} AIC calculation that takes on the minimum value (among all the AIC values).

55. If m is the index associated with the best model, we can define:

$$m = \arg\min_i \{AIC_i\}, \text{ and}$$

$$\Delta_i = \text{AICc}_i - \text{AICc}_{min} \tag{16}$$

56. This transformation forces the best model to have $\Delta = 0$, while the rest of the models have positive values. The value Δ_i provides a quick *strength-of-evidence* comparison of candidate models where the larger the value of Δ_i, the less plausible the i^{th} model is (among the candidate set).

26 The correction refers to the additive term 2·k·(k+1)/(n-k-1), that should be used when n / k > 40.

27 To be precise, it estimates the Kullback-Leibler discrepancy.

57. The rankings of the candidate models (that is, their relative distance from the "best" model) are also presented in this paper. The rankings are based on the scientific literature,[28] which suggests a simple rule of thumb that is useful to assess the relative merits of models in the proposal set. This "rule of thumb" is shown in Table A.10.

Support of a given model on the basis of the Δ_i distance according to the AICc

Table A. 10

Δ_i	Level of empirical support for model i
0 – 2	Substantial
4 – 7	Considerably less
> 10	Essentially none

58. Table A.2 shows that the power of a model with a value Δ_i that is within two units from the best model, is similar to the best model. However, when $\Delta_i > 10$, the i^{th} model is much worse than the best model, and has to be excluded from the eligible ones.

59. The BIC, which is based on Bayesian theory, was introduced by Gideon Schwarz (1978) as a competitor to the AIC. The BIC is linked to AICc formally, but not theoretically. Schwarz developed the BIC to serve as an asymptotic approximation to a transformation of the Bayesian posterior probability of a candidate model. In large-sample settings, the fitted model favoured by BIC ideally corresponds to the candidate model which is *a posteriori* most probable, ie the model which is rendered most plausible by the available data. The computation of BIC is based on the empirical log-likelihood and does not require the specification of priors.

60. The general expression for the BIC is given in (17).

$$BIC_i = -2 \cdot \ln(\mathcal{L}(\eta|\text{data})) + k \cdot \ln(n) \tag{17}$$

where η is the maximum likelihood estimation of model parameters, n is the number of observation, k is the number of parameters, $\mathcal{L}(\cdot|\text{data})$ is the likelihood of the function, ln is the natural logarithm, i is the i^{th} model (or PI) for which the BIC quantity is computed.

61. It can be shown that:[29]

$$\text{Prob}\{\text{Model h} \mid \text{Observations}\} = \frac{\exp\{-\delta_i/2\}}{\sum_j \exp\{-\delta_j/2\}} \propto \exp\{-\delta_i/2\}, \tag{18}$$

where $\delta_i = BIC_i - BIC_{m^*}$, and $m^* = \arg\min_i \{BIC_i\}$, is, as in the AIC, the index associated to the best model, or in other words the model with the lowest BIC measure. In terms of BIC criterion, *strength-of-evidence* means, substantially, posterior probability of model i being true given the observed data.

62. Criteria and rules of the thumb similar to the AIC's may be used in the case of BIC to rank models with respect to the best one:[30]

[28] See K Burnham and D Anderson, "Multimodel inference: understanding AIC and BIC in model selection", *Sociological Methods and Research*, vol 33, no 2, November 2004, p 271.

[29] Given R competing models, we assume for each of them an a-priori probability of 1/R.

[30] See, for example, R Kass and A Raftery, "Bayes factors", *Journal of the American Statistical Association*, vol 90, 1995, p 777.

Support of a given model on the basis of the δ_i distance according to the BIC

Table A. 11

δ_i	Evidence against i
0–2	Not worth more than a bare mention
2–6	Positive
6–10	Strong
> 10	Very strong

63. When fitting models, it is always possible to increase the likelihood by adding parameters, but this may result in over-fitting. BIC and AICc resolve this problem by adding the penalty terms given in (19).

$$2 \cdot k + 2 \cdot k \cdot (k+1)/(n-k-1), \text{ for the AICc,}$$

$$k \cdot \ln(n), \text{ for the BIC} \qquad (19)$$

64. From the above, we can show that BIC > AICc if and only if $n > \exp\{2+2\cdot(k-1)\cdot(n-k-1)^{-1}\}$. In a realistic case n >> k, and the second term can be approximated with $e^2 = 7.3891$; so, in general, the BIC is more stringent than the AICc because it measures the discrepancy in a more prudential way. Applying this to our estimation we expect smaller AICc measures relative to the BIC ones.

65. As mentioned earlier, the use of the AICc and BIC goodness-of-fit measures with OLS estimates requires a Gaussian structure to be found in the residual $\{\varepsilon_h\}$ of the non-linear regressions.

66. To compute the best Gaussian structure compatible with the framework, we adopt a "minimum distance strategy", based on the P-value criterion. In other words, the distance from the empirical distribution of the residuals and a normal distribution is investigated and measured according to the P-value of a Kolmogorov-Smirnov test (at a significance level of 10%). We prefer to adopt a "distance" criterion rather than a "moment" matching (ie the comparison between theoretical and empirical moments of different order, for instance the mean and the standard deviation), as the latter is a less robust criterion for assessing the existence of normal distributions compatible with the residuals.[31]

67. Analytically, if $F_{N(\mu,\sigma)}$ is a generic normal distribution we look for $(\mu*,\sigma*)$ multi-parameter such that:

$$(\mu*,\sigma*) = \text{Arg Max } S(d(\hat{F}_n, F_{(\mu,\sigma)})) \qquad (20)$$

where $\hat{F}_n$ is the empirical distribution function of residuals $\{\varepsilon_i\}$, d is an appropriate random distance between $\hat{F}_n$ and $F_{N(\mu,\sigma)}$, and S is the survival function of d.

68. It is important to recognise that if S is a strictly countermonotonic function of d, S is the maximum when d is the minimum, and the related optimisation problem is equivalent to (21).

[31] The "moment" matching criterion checks whether or not the residuals come from a normal distribution with moments equal to the empirical ones. The "distance" criterion checks any moments that deviate from the empirical distribution of the residuals. The "distance" criterion allows for comparison of the normality condition along the residuals curve and not only at specific points (the "moment matching" criterion does the latter).

$$(\mu^*,\sigma^*) = \mathrm{Arg\,Min}\, d(\hat{F}_n, F_{(\mu,\sigma)}) \tag{21}$$

69. For our purposes, it is useful to represent the distance d by the Kolmogorov-Smirnov statistic.

70. We want R(x) to be an unbiased estimator of the OpCaR, so we impose the constraint $\mu^*=0$. The normal distribution therefore becomes $F_{N(0,\sigma)}$.

71. $F_{(0,\sigma^*)}$ is the Gaussian model with minimum distance to $\hat{F}_n$, and S(d) is the P-value linked to the null hypothesis H_0: $\hat{F}_n = F_{(0,\sigma^*)}$. Therefore, when S(d) is sufficiently high, we can affirm that the minimum distance Gaussian model $F_{(0,\sigma^*)}$ can, in general, represent the observed model $\hat{F}_n$.

72. Based on the description above, the steps of the procedure can be summarised as follows:

- Let ε_i be the residuals of the referred indicator PI_i. First, the empirical mean, m_{ε_i}, and standard deviation, s_{ε_i}, of the ε_i are computed as shown in (22).

$$\begin{aligned} m_{\varepsilon_i} &= \frac{\sum_{j=1}^{n} \varepsilon_j}{n}, \\ s_{\varepsilon_i} &= \sqrt{\frac{\sum_{j=1}^{n} [\varepsilon_j - m_{\varepsilon_i}]^2}{n-1}} \end{aligned} ; \tag{22}$$

- Let $\xi_{\varepsilon_i} = \ln(s_{\varepsilon_i})$ and denote by $\hat{F}_h$ the empirical distribution function of the $\{\varepsilon_i\}$;
- The following optimisation problem is solved (if a solution exists),

$$S(\sqrt{n} \cdot D_n(0,\xi_{\varepsilon_i})) = \max \text{ with respect to } \xi_{\varepsilon_i}, \tag{23}$$

where D_n is the Kolmogorov-Smirnov statistic that is shown in (24).

$$D_n(0,\xi_{\varepsilon_i}) = \mathrm{Sup}_{\varepsilon} \left| \hat{F}_i(\varepsilon) - \Phi\left(\frac{\varepsilon}{\exp\{\xi_{\varepsilon_i}\}} \right) \right| \tag{24}$$

and S is the survival function of the scaled K-S statistic.

73. It is assumed that the starting point for the iterative procedure that solves the above unconstrained problem is ${\xi_{\varepsilon_i}}^0 = \ln(s_{\varepsilon_i})$.

74. If $\xi^*_{\varepsilon_i}$ is the solution of the unconstrained problem, then $S(\sqrt{n} \cdot D_n(0,\xi^*_{\varepsilon_i}))$ is the P-value linked to the null hypothesis H_0: $\varepsilon_i \sim \mathrm{Normal}(0,\sigma^*_{\varepsilon_i})$, with $\sigma^*_{\varepsilon_i} = \exp\{\xi^*_{\varepsilon_i}\}$. If the P-value – which is the maximum by construction – is sufficiently high, we can confirm the null hypothesis H_0 and consider the best suitable Gaussian structure behind the ε_i as an outright representation of the residuals structure.

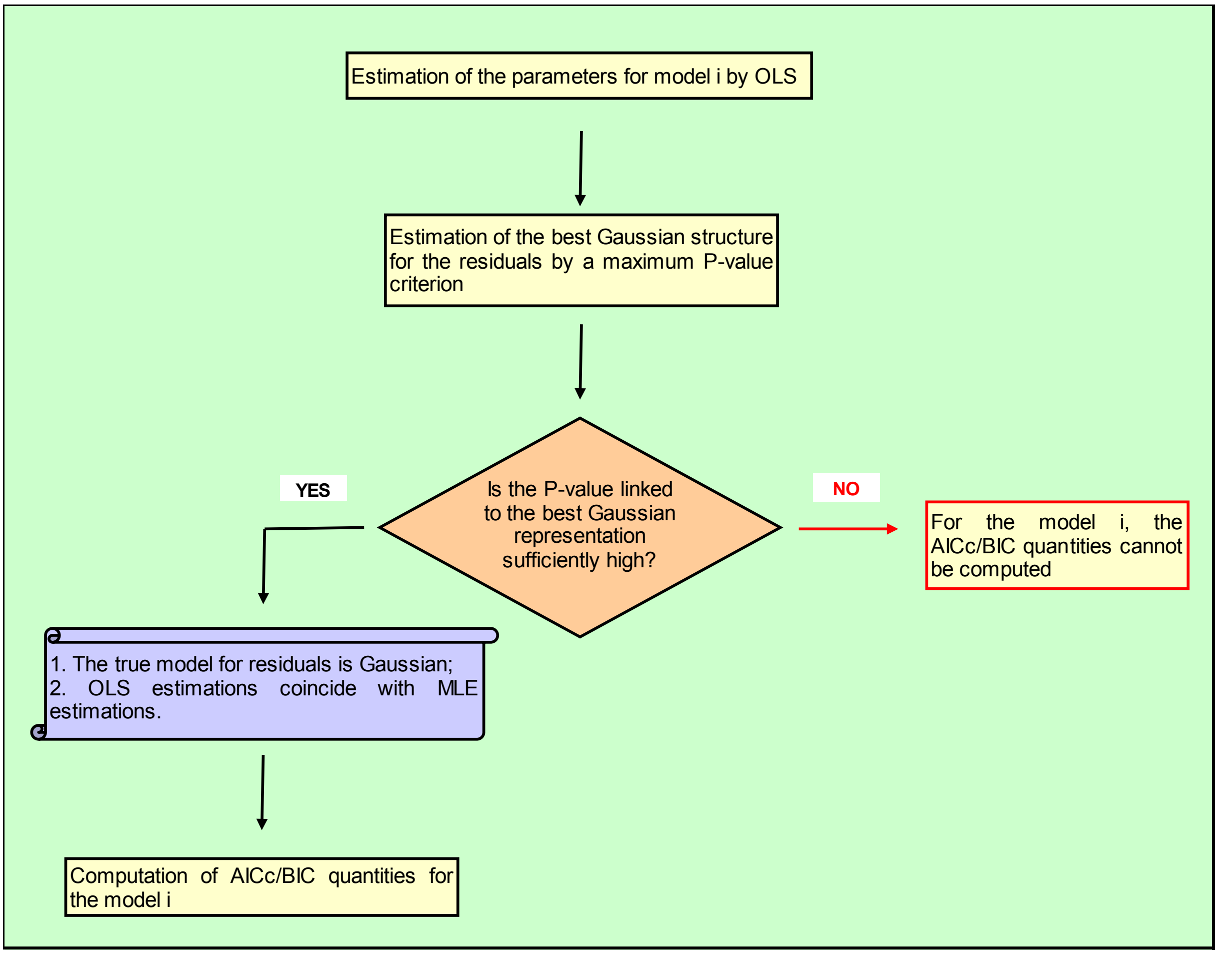
75. For computational purposes, we adopt the analytical approximation[32] for S that is shown in (25).

[32] See N Johnson and S Kotz, *Distributions in Statistics – Continuous Univariate Distributions 2*, John Wiley & sons, New York, 1970, p 255.

$$S(z)=\begin{cases}1-\dfrac{\sqrt{2\cdot\pi}}{z}\cdot\exp\left\{-\dfrac{1}{8}\left(\dfrac{\pi}{z}\right)^2\right\} & z\le 1\\ 2\cdot\exp\left\{-2\cdot z^2\right\} & z>1\end{cases} \tag{25}$$

76. The logical sequence of the described steps to get AICc and BIC statistics is summarised in the following workflow diagram.

Figure A.3: Workflow of the procedure for the computation of the AICc and BIC for any PI



5. The results of the non-linear regressions

77. The process described in Section III.3 was applied to the regression data set of 89 banks, as identified in Table A.9. For each indicator, after getting the parameters estimates, the normality of the residuals was assessed, considering that, as noted previously, this is a necessary condition for the use of the AICc and BIC with OLS estimates. Following the procedure explained in the previous section, the empirical distribution of the residuals was compared with a normal distribution and the P-Value of the Kolmogorov-Smirnov test at a significance level of the 10% computed.

78. However, by construction, the P-value is a hypothesis test that is not informative in terms of goodness-of-fit between the dependent and independent variables. It simply says whether or not the

indicator is non-linearly linked to the OpCaR and does not give indications in terms of accuracy of the indicator in representing the OpCaR itself.[33]

79. Therefore, for those variables that passed the P-value test, the AICc and BIC measures were computed. As described in the previous section, these are appropriate goodness-of-fit statistics for non-linear regressions. The lower the values of the statistics the better; and when an indicator exhibits AICc or BIC figures larger than 10 units from the best performing indicator, this means that it behaves very poorly with respect to the best one.

80. The results showed the significant superiority of the BI over all the investigated indicators in capturing a bank's operational risk exposure. Indeed, any other indicator had AICc and BIC statistics larger than 30 units from the value of the BI, where 10 units of distance are already sufficient to qualify these indicators as unfit in representing a given phenomenon when compared with the BI.

81. To understand the reasons for the superior performance of the BI over the other indicators, and in particular over the GI, an in-depth investigation on the underlying components was undertaken. In particular, the interest, services and financial components of the BI and GI indicators were isolated and their explanatory power separately investigated according to the same procedure adopted for the indicators.[34]

82. From the analysis, the following remarks can be drawn:

- The use of the "nNet" basis (ie income – expenses) is crucial to enable the "interest" component to capture a bank's operational risk;
- The use of the "full" allows for a significant increase in the risk sensitivity of the component "services", in the light of the relevant decrease of the BIC and AICc statistics;
- The use of an "absolute" figure and the inclusion of the "banking book" make the "financial" component much more powerful with respect to how it is treated within the GI (that is as "net" and with the "trading book" only).

83. The analysis of the components can suggest a different approach to selecting the best indicator. Following a bottom-up process, one can identify the components with highest explanatory power and then aggregate them to build the best indicator. Here, if the components are not negatively correlated, an indicator built on the best performing components should also be the best as a whole.

84. Looking at the interest, services and financial components and aggregating those with best performance, one gets back exactly the BI. This reverse-engineering exercise, therefore, justifies the use of the BI as the (best) proxy for operational risk regulatory capital.

[33] If several indicators have P-values above the significance level, this means that all of them are non-linearly related to a bank's OpCaR. However the indicator with largest P-value is not necessarily the best one to capture a bank's operational risk. An indicator might be at the same time non-linearly related to a bank's OpCaR and very poor in representing/predicting the OpCaR itself.

[34] The explanatory power of the BI was substantially unaffected by the inclusion or exclusion of the dividend as this component showed low sensitivity to a bank's operational risk. Therefore, the dividend component might be included into the BI perimeter, if this could ease the implementation of the indicator, and this would not affect the properties of the indicator.

Appendix

Pros and cons of major proxies investigated in the analysis

	Pros	Cons
Gross income	This is a simple and practical measure, and easy to discuss with senior management. It has already been applied in Basel II.	It is a less effective predictor than the BI of a bank's operational risk. It misses or nets items that are related to operational risk loss, such as other operating expenses or fee and commission income/expenses. GI and its components (eg trading income) could take a negative value, resulting in a reduction of the regulatory capital while operational risk may arise. Some its components (eg trading income) are rather volatile over time.
Business indicator	It takes into consideration elements that GI does not, such as other operating expenses and the volume of fee and commission business, and thus it measures the volume of activities better. It or its components never take a negative value, hence inconsistent and counterintuitive effects on the regulatory capital are prevented. It has a statistically stronger explanatory power than other proxy indicators. It is robust to extreme variations over time. It remains simple and easy to implement.	It shows the same issues as GI for banks with high interest margins.
Total assets	It is aligned with a bank's size and potentially able to reflect the volume of business. It is relatively stable over time.	It is a less effective predictor than the BI of a bank's operational risk. Some banks' business activities would not be reflected in an asset-based indicator. Using total assets as proxy indicator raises the possibility of overlapping with the Leverage ratio.
Admini-strative costs	Like total assets, administrative costs are aligned with a bank's size and potentially able to reflect the volume of business. A costs-based indicator is relatively stable over time.	It is a less effective predictor than the BI of a bank's operational risk. A costs-based indicator would perform in a counter intuitive manner and would drive negative behaviours (reducing the expenses and investments on processes and systems while seeking to maintain transaction volumes, would result in an increase in operational risk).
Total provisions	It covers a certain range of bank's operating activities. It includes provisions for operational risk events and losses.	It is a less effective predictor than the BI of a bank's operational risk. It encompasses many different components and is very hard to find a common, universally accepted definition for this indicator. Its use for regulatory purposes would therefore be highly exposed to interpretative issues and inconsistencies during its implementation.

Annex 4: Loss data collection and risk management guidance for large internationally active banks under the revised SA for operational risk

The following describes loss data collection and operational risk management qualitative standards that should be observed by large internationally active banks under Pillar 2:

A. Loss data collection

1. Relevant operational risk data should be systematically collected and tracked, including material losses by business activity and event type.

2. Following an operational risk event, the bank should be able to separately identify gross loss amounts, insurance recoveries, and other recoveries, except when losses are rapidly recovered. For this purpose, the bank should adopt clear and consistent definitions of "gross loss", "insurance recoveries" and "recoveries except insurance".

3. Appropriate thresholds for loss data collection should be implemented, based on gross loss amounts. Collection threshold(s) should not lead to the omission of loss event data that are material for effective management and assessment of operational risk.

4. For each operational loss event, the bank should be able to identify the date of the discovery, the date of accounting or reserve and the date of recovery (if applicable), and should try to identify the date of occurrence.

5. The bank should identify and gather information regarding near misses, operational risk gains, and opportunity costs/loss revenues, as well as operational risk contribution to credit risk and market risk and other data that may provide useful operational risk management information.

6. The bank should document and implement procedures for assessing the ongoing relevance of internal loss data. For example, the bank should consider the appropriate retention period for data used to estimate model parameters, and the appropriate use of historical loss data relating to divested business activities.

7. For risk management and assessment purposes, the losses caused by a common operational risk event or by multiple events linked to a single root-cause should be grouped together.

8. The internal loss data collection process should be subject to regular independent review by internal and/or external audit functions.

9. Losses should be mapped against the loss event type categories set out in the Basel framework (Annex 9). The following summary information should be reported to the competent supervisor in a timely manner:

(i) Number of loss events above the threshold

(ii) Total loss amount above threshold

(iii) Maximum single loss

(iv) Sum of five largest losses

B. Operational risk management

1. The bank should have an operational risk management framework that is conceptually sound and implemented with integrity.

2. The bank should have robust processes for managing operational risk throughout its business. As part of the internal validation process, a bank should assess the appropriateness of its risk management framework and the effectiveness of its implementation in order to ensure that the framework remains "fit for purpose" and operates as the board and senior management intend.

3. The bank's board and senior management should have responsibility for approving material aspects of the overall operational risk framework. They should understand how operational risk affects the bank and comprehend the management reports submitted to them. The material aspects of the overall operational risk framework include:

(i) Policies, procedures, and organisational structures – including established responsibilities and accountabilities—for managing operational risk;

(ii) Statement of risk appetite and/or thresholds or levels of acceptable risk;

(iii) Activities to identify, assess, measure, monitor, and control or mitigate operational risk; and

(iv) Incentives to improve the management of operational risk throughout the firm.

4. The operational risk management function must be independent of the business lines and other functions that incur risk (eg finance).

5. The bank should assign sufficient resources to managing operational risk effectively in major business activities, as well as to undertake appropriate control and audit activities.

6. The bank's risk management framework should include mapping and reporting of gross income and loss experience by business line or activity.

7. The bank's operational risk management function should be responsible for coordinating: the development of strategies to identify, assess, monitor and control/mitigate operational risk; the codification of firm-level policies and procedures concerning operational risk management and controls; the design and implementation of the firm's operational risk assessment methodology; and the design and implementation of a risk-reporting system for operational risk.

8. The bank's assessment of operational risk should be closely integrated with the firm-wide risk management processes. Operational risk framework reporting must be an integral part of the process of monitoring and controlling a bank's risk profile. In its assessment of operational risk, a bank should include risk inherent in new areas (products, activities, processes, and systems) and ensure that the bank's risk profile is updated regularly.

9. The bank should regularly report its operational risk exposures, including material operational losses, to business unit management, senior management, and to the board of directors. The reporting should be actionable and support decision making.